1. Consider a droplet that forms through the process of homogeneous nucleation. Starting with the expressions for bulk thermodynamic energy and surface energy, derive expressions for the change in Gibbs free energy (eq. 3.9), the critical radius (eq. 3.12) and the critical energy barrier (eq 3.16).

\[ \Delta G = -\frac{4}{3} \pi n \sigma V R^3 \ln \left( \frac{e}{e_s} \right) + 4 \pi \sigma V R^2 \]  

3.9

\[ R^* = \frac{2 \sigma V}{n \sigma V \ln(e/e_s)} \]  

3.12

\[ \Delta G^* = \frac{16 \pi \sigma V^3}{3 \left[ n \sigma V \ln(e/e_s) \right]^2} \]  

3.16

Clearly explain what the bulk thermodynamic energy and surface energy expressions represent. Include explanations of the steps you take in the derivation, not just the equations! What do \( \Delta G \), \( R^* \), and \( \Delta G^* \) represent, physically?

2. What are the three desirable characteristics of thermodynamic diagrams? Describe and discuss the similarities and differences between a Tephigram and Skew T-log p diagram.

3. a) Calculate the variation of temperature and supersaturation (in 0.2 cm intervals) within a thermogradient diffusion chamber that is 1 cm deep. Assume:

(1) \( T_B = \) temperature of bottom plate = 22 °C
    \( T_T = \) temperature of top plate = 27.5 °C

(2) \( T_B = 22^\circ\text{C} \)
    \( T_T = 30^\circ\text{C} \)

Use Teten’s formula to compute the saturation vapor pressure as a function of temperature for (1) and (2):

\[ e_s(T) = 6.11 \exp \frac{17.67T}{T + 243.5} \]

where \( e_s \) is in mb and \( T \) is in degrees Celsius. This approximation is accurate to within 0.3% in the range -35°C \( \leq T \leq 35°C \).
b) Assume the concentration of particles activated at any given supersaturation is given by:

\[ N_{\text{CCN}} = cS^k \]

where S is in percent.

Assume:

(3) \( c=800 \text{ cm}^{-3}; k=0.5 \)
(4) \( c=200 \text{ cm}^{-3}; k=0.7 \)

Calculate the concentration activated for both (3) and (4) using the chamber temperatures given in (1) at EACH 0.2 cm interval. What types of air masses would you expect to have the activation profiles given by (3) and (4)?

4. a) Define “aerosols” and the three categories they are divided into.
   b) For each category defined in part a, discuss in detail the sources and sinks for aerosols in that category. Why do “large” aerosols have the longest lifetimes?
   c) Which sizes and types of aerosols are most likely to serve as CCN? How do observations coincide with theory?