Selective ion charging of droplets in thunderstorms under arbitrary oriented electric field

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ABSTRACT

Coupling of an aerosol particle modelled as an ideal conducting sphere to the moving ionized gas has been investigated in the presence of an arbitrarily directed external electric field. Attachment coefficients for ions have been found analytically and numerically including the important case when the electric field $\mathbf{E}_\infty$ and gas flow velocity $\mathbf{U}_\infty$ are not collinear. Trajectory approach has been used. The stationary charge of the individual isolated aerosol as function of angle between $\mathbf{E}_\infty$ and $\mathbf{U}_\infty$ has been examined.

INTRODUCTION

The selective ion capture (**SIC**) charging mechanism was concurred by Wilson, (1929). He clearly understood the basic principle of "slow" ion rearrangement in the vicinity of the moving particle. Later Pouthenier and Moreau-Hanot (1932) calculated a simple dynamic of the unipolar charging in the external uniform electric field. The maximum (stationary) charge acquired by the isolated spherical particle, q_U, is determined only by electric field and particle's radius: $q_U = 3E_\infty R^2$. **SIC** mechanism differs from the "symmetrical" charging in the strong electric fields, when $E_\infty B_i > U_p$, which was investigated in detail by Drukarev (1946). In this case the charging current does not depend on direction of vectors $\mathbf{U}_p$ and $\mathbf{E}_\infty$. And particle acquires a nonzero charge if only ion conductivities are different: $\sigma_{i+} \neq \sigma_{i-}$. Much later Sumiyoshitani (1996) has analyzed how the gas flow field, the electric field and polarized forces influence ion movement in different coordinate systems. The numerical analysis of ions trajectories has been performed but from these preliminary calculations it is unclear how the ions current and the stationary charge on the particle depend on the angle between the electric field and the gas flow. The basic goal of this short paper is to calculate charging rates and stationary charge for isolated aerosol with any angle α between $\mathbf{E}_\infty$ and $\mathbf{U}_\infty$ in trajectory assumption.

PROBLEM STATEMENT: BASIC EQUATIONS AND TRAJECTORY APPROACH

Trajectory approach means that we neglect ion diffusion and recombination processes and consider ions as long lived particles. Also we do not take into account a uncompensated space charge in the vicinity of the particle and assume that a result field is a sum of external and Coulomb terms. This suppositions produce a simple equation:

$$\frac{\partial n_{i\pm}}{\partial \tau} + \mathbf{V}_\pm(\mathbf{r},\tau)\nabla n_{i\pm} = \frac{dn_{i\pm}}{d\tau} = 0 \tag{1}$$

It means that ions concentration is constant along each trajectory and is determined by its magnitude at the start point of each track. Ion trajectories configuration, $\mathbf{F}(r,\theta,\phi)$, are described by the following system of differential equations:

$$\frac{dr}{V_{r\pm}(r,\theta,\phi)} = \frac{rd\theta}{V_{\theta\pm}(r,\theta,\phi)} = \frac{r\sin\theta d\phi}{V_\phi(r,\theta,\phi)} = d\tau, \tag{2}$$

Equations (2) are difficult to solve in the whole space around the aerosol, for any r, θ, ϕ and α. It is simple to do for the case $\alpha = 0$ or $\alpha = \pi$, since $V_\phi = 0$ and the system (2) reduces to the one equation. As it follows from the computer mapping of the trajectories for the several α and calculation of the current, there is a clear difference between the potential and the Stokes flow. It appears in the form of bounds determining the region G_a (collision cross-section, see Fig.1), and as consequence - in the magnitudes of the attachment coefficients and ion current. $\mathbf{V}_\pm$ is the normalized on U_∞ ion velocities which components are determined via flow functions Ψ:

$$V_{r\pm}(r,\theta,\phi) = \frac{1}{r^2}\{(\cos\theta\cos\alpha + \sin\theta\sin\alpha\cos\phi)\Psi_0 + \cos\theta\Psi_{1\pm} + \eta_\pm\}; \tag{3}$$

$$V_{\theta\pm}(r,\theta,\phi) = -\frac{1}{2r}\{(\sin\theta\cos\alpha - \cos\theta\sin\alpha\cos\phi)(\Psi_0)'_r + \sin\theta(\Psi_{1\pm})'_r\};\;\; V_\phi(r,\theta,\phi) = -\frac{1}{2r}\sin\alpha\sin\phi(\Psi_0)'_r; \tag{4}$$

where

$$\Psi_{1\pm} = \xi_\pm(r^2 + \frac{2}{r}),\ \Psi_0 = \begin{cases} r^2(1 - \frac{3}{2r} + \frac{1}{2r^3}), & \text{Stokes flow} \\ r^2 - \frac{1}{r}, & \text{potential flow} \end{cases},\ \eta_\pm = \frac{qB_{i\pm}}{R^2 U_\infty},\ \xi_\pm = \frac{B_{i\pm}E_\infty}{U_\infty} \quad (5)$$

r is the radial spherical coordinate normalized on particle radius,R, ϕ is the polar angle, θ is the azimuth, α is the angle between the vectors $\mathbf{E}_\infty$ and $\mathbf{U}_\infty$, which are lying in XZ plane, as is shown in Fig. 1, $B_{i\pm} = \pm|B_{i\pm}|$, E_∞ is the magnitude of the electric field $\mathbf{E}_\infty$ which is always assumed to be oriented along z- axis: $E_z = E_\infty$.

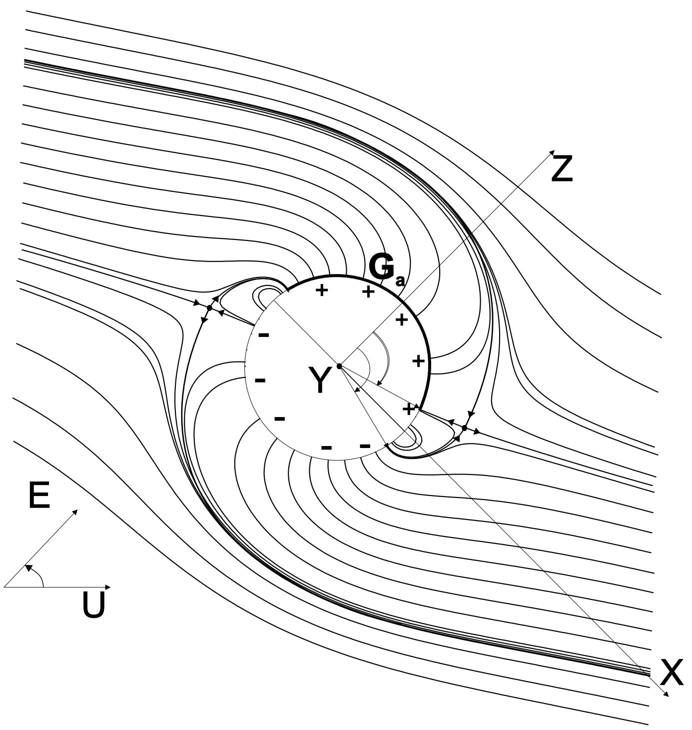

Figure 1: Negative ion trajectories, $Y = 0$ plane, $\alpha = 0.25\pi$, $|\xi_-| = 0.25$, $q = 0$.

This configuration of the electric field and gas flow vectors is chosen only for simplification of numerical calculation, since $\mathbf{E}_\infty$ does not depend on ϕ. And the curve on the sphere is determined by equation $E_r(\theta) = 0$ has fixed coordinates $\theta = \theta_0$ and does not depend on ϕ also for any α values, $\cos\theta_0 = -q/q_U$.

In Fig. 1, a system of trajectories at the aerosol is presented for the Stokes flow in plane XZ. The structure of trajectories varies from the symmetric case $\alpha = 0$ - and the current determined by the outer trajectories is not equal to zero here.

NUMERICAL RESULTS

To obtain ions current the following numerical procedure has been performed. The equation (2) has been solved under given boundary condition on the surface of the particle. Hence, electric field direction does not change, the range $0 < \theta < \theta_0$ has been used as a set of start trajectory points for the positive ions and $\theta_0 < \theta < \pi$ for the negative one, for $0 \le \alpha \le \pi/2$. Each track has been computed from the start to the end point. If ion goes to infinity (e.g. calculated end point $r = 15$) it means, that this ion makes a contribution to the total current. If ion is attached to the particle in the conjugate point ($r = 1$, $\theta = \theta^*$) - no contribution to the total current occurs: this ion path is closed. But the number of ions on the closed tracks is incredibly small in our assumption.

The algorithm determines G_a (distinguishing closed and semi-infinite trajectories) and performs an integration of $3E_\infty \cos\theta$ over all G_a area. Interesting results are presented in Fig.2. left) and show ion current to neutral particle. The clear difference between Stokes and potential flow is a presence of the cutoff angle α_c for the first one. This angle is a function of $\xi_\pm$ and for the case $|\xi_\pm| \ll 1$ both flow regimes have a same current shape: $\tilde{J} = \sin(\alpha)$. All currents here and after are normalized to the value $J_{0\pm} = 0.25 q_U \nu_{i\pm}$, where $\nu_{i\pm} = 4\pi\sigma_\pm$. As it follows from the numerical analysis, both currents can be approximated as:

$$\tilde{J}_- = \begin{cases} \sin\left(\frac{\alpha}{\alpha_{c-}}\frac{\pi}{2}\right), & 0 \le \alpha < \alpha_{c-} \\ 1 & \alpha_{c-} \le \alpha \le \pi, \end{cases} \quad \tilde{J}_+ = \begin{cases} 1 & 0 \le \alpha < \pi - \alpha_{c+}, \\ \sin\left(\frac{\alpha}{\alpha_{c+}}\frac{\pi}{2}\right), & \pi - \alpha_{c+} \le \alpha \le \pi \end{cases} \quad (6)$$

the cutoff angle α_{c-} can be written as:

$$\alpha_{c\pm} = \frac{\pi}{2}(1 - |\xi_\pm|)^s,\ s \approx 1.8 \quad (7)$$

Also the same approach has been used for the current calculation for charged cloud particle. Only for the case

$|\xi_\pm| \ll 1$ it is possible to approximate numerical curves (Fig. 2. right) by the following expressions

$$\tilde{J}_+ = \begin{cases} -4\tilde{q} + (1+\tilde{q})^2, & 0 \le \alpha < \frac{\pi}{2} \\ -4\tilde{q} + \sin\alpha(1 + \frac{\tilde{q}}{\sin\alpha})^2, & \frac{\pi}{2} \le \alpha \le \pi - \alpha_t \\ -4\tilde{q},\ (-1 \le \tilde{q} \le 0) & \pi - \alpha_t \le \alpha \le \pi, \\ 0,\ (0 \le q) & \pi - \alpha_t \le \alpha \le \pi, \end{cases} \quad \tilde{J}_- = \begin{cases} 4\tilde{q},\ (0 \le q) & 0 \le \alpha \le \alpha_t, \\ 0,\ (-1 \le \tilde{q} \le 0) & 0 \le \alpha \le \alpha_t, \\ 4\tilde{q} + \sin\alpha(1 - \frac{\tilde{q}}{\sin\alpha})^2, & \alpha_t \le \alpha \le \frac{\pi}{2} \\ 4\tilde{q} + (1-\tilde{q})^2, & \frac{\pi}{2} \le \alpha < \pi \end{cases} \quad (8)$$

where $\sin\alpha_t = |\tilde{q}| = |q|/q_U$. For the full range of the parameter $|\xi_\pm|$: $0 < |\xi_\pm| < 1$, there is no analytical solution $\tilde{J}_\pm(\alpha, \tilde{q}, \xi_\pm)$ exists. If $|\overline{q}| > 1$ then ion currents do not depend on α and $\xi_\pm$ and are expressed by the following well-known formulae

$$\tilde{J}_+ = \begin{cases} -4\tilde{q}, & \tilde{q} < -1; \\ 0, & \tilde{q} > 1; \end{cases} \quad \tilde{J}_- = \begin{cases} 0, & \tilde{q} < -1; \\ 4\tilde{q}, & \tilde{q} > 1; \end{cases} \quad (9)$$

The general equation for the charge dynamics of an isolated cloud particle is

$$\frac{dq}{dt} = J_+(q, E_\infty, \alpha) + J_-(q, E_\infty, \alpha) \ \text{ or } \ \frac{d\tilde{q}}{dt} = \nu_{i+}\tilde{J}_+ - \nu_{i-}\tilde{J}_- \quad (10)$$

It is easy to find the stationary charge from the condition $dq/dt = 0$. Thus, we should resolve the equation $J_+ + J_- = 0$ in respect to q. For currents $\tilde{J}_+$, $\tilde{J}_-$ this equation should be written as $\tilde{J}_+ - \tilde{J}_- = 0$, because $\tilde{J}_\pm$ are always positive. Substituting the currents from formulae (8)to the equation above and finding roots of the quadratic equations for the four values of the α region, the stationary charge on the aerosol $\tilde{q}_s = q_s/q_U$ can be written as

$$\tilde{q}_s = \begin{cases} \frac{1}{p}(p + 2 - 2\sqrt{p+1}), & \alpha \le \alpha_t \\ \sqrt{\sin\alpha}\frac{\sqrt{p} - \sqrt{\sin\alpha}}{1 + \sqrt{p\sin\alpha}} & \alpha_t < \alpha \le \frac{\pi}{2} \\ \sqrt{\sin\alpha}\frac{\sqrt{p\sin\alpha} - 1}{\sqrt{p} + \sqrt{\sin\alpha}} & \frac{\pi}{2} < \alpha \le \pi - \alpha_t^* \\ p(-(\frac{1}{p} + 2) + 2\sqrt{\frac{1}{p} + 1}), & \pi - \alpha_t^* \le \pi \end{cases} \quad (11)$$

where

$$\sin\alpha_t = \frac{1}{p}(p + 2 - 2\sqrt{p+1}),\ \sin\alpha_t^* = p(\frac{1}{p} + 2 - 2\sqrt{\frac{1}{p} + 1}) \quad (12)$$

For the characteristic time $\sim 5/\nu_i$ the charge of the particle will become stationary and will be determined by the formulae above. The charge q_s in the case $B_+ = |B_-|$ is equal to 0 if $\alpha = \pi/2$ and it is rising up by the absolute value if $\alpha \to 0$. Obviously, α is not equal to angle α' between $\mathbf{E}_\infty$ and $\mathbf{g}$, because $\mathbf{U}_p \not\parallel \mathbf{g}$, where $\mathbf{U}_p$ is determined from the Newton's law of motion. The connection between α and α' be as following:

$$\cos\alpha = -\frac{\cos\alpha' + e_g}{\sqrt{1 + 2e_g\cos\alpha' + e_g^2}},\ e_g = \frac{q|\mathbf{E}_\infty|}{m|\mathbf{g}|} \quad (13)$$

The minus sign is due to $\mathbf{U}_\infty = -\mathbf{U}_p$, so in case $e_g \ll 1$ we have $\alpha = \pi - \alpha'$. In the opposite case $e_g \gg 1$, evidently, $\mathbf{U}_\infty$ is directed along $\mathbf{E}_\infty$ if $q < 0$ and has an opposite direction in case $q > 0$. The wind velocity and media characteristics (viscosity, e.g.) are not included into result (13) because the stationary case has been considered.

CONCLUSIONS

The system considered works as a distributed ElectroHydroDynamic (EHD) generator, with charge separation and rearrangement in the vicinity of the particle surface. The **SIC** mechanism of charging a conducting cloud particle is the result of the ion trajectories splitting at the aerosol's surface. If vectors $\mathbf{E}_\infty$ and $\mathbf{U}_\infty$ are not collinear, then the symmetry in respect to the angle ϕ is violated and the boundary of region G_a is determined then as function of ϕ. In case of weak electric fields, it is possible to find the current in the whole range of angles α, and the significant thing is that therewith the expression for the current will not depend on the flow regime. On the other hand, if

$\xi = 0.3$, for example, then according to the calculated curves, the difference in the regimes will be considerable. The basic effects are as following:

1. Currents for potential and the Stokes flow differ, while for the case $\mathbf{E}_\infty || \mathbf{U}_\infty$ it is the same. The magnitude of this difference rises when $|\xi_\pm|$ rises from 0 to 1. For the case of fast ions, $|\xi_\pm| > 1$, the angle α does not play the role.

2. Whereas angle α rises from 0 (or decreases from π), the total ion current remains the same until angle reaches the threshold value α_t (or $\pi - \alpha_t$). In other words, the ion currents in case of $0 < \alpha < \alpha_t$ for negative ions and $\pi - \alpha_t^* < \alpha < \pi$ - for positive one remain unchanged. This "stability" is proportional to the particle charge, see Fig. 2. right).

3. One more important peculiarity consists in the fact that in case of the Stokes regime, the current (for the negative ions, e.g.) achieves its maximum at the angles smaller than $\pi/2$, namely $\alpha = \alpha_c$, which occurs, however, only in case of the fields close to the boundary limit $1 > |\xi_\pm| > 0.1$.

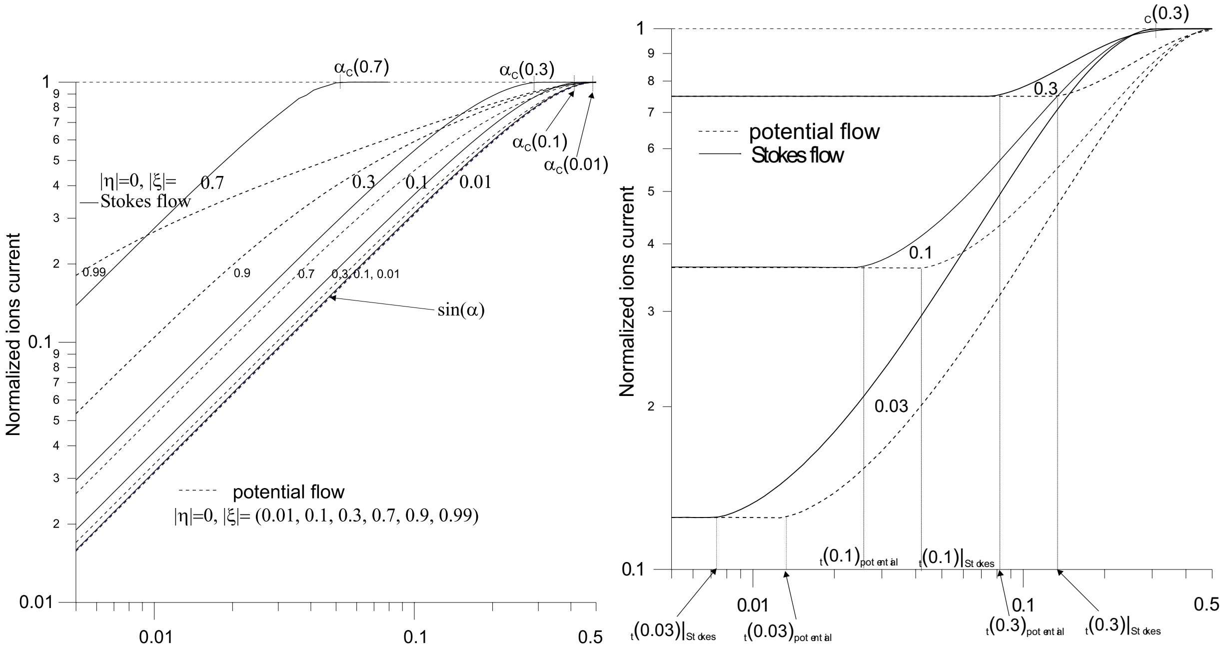

Figure 2: **left)** Negative ion current as function of α, $q = 0$. **right)** Negative ions current, normalized by the value $\tilde{J}_- = (1 + \tilde{q})^2$, as function of α, $q \neq 0$. Solid lines are for the Stokes flow, dashed - for the potential flow.

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