Charge Spectra of Colliding Ice Crystals and Hailstones

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ABSTRACT

The 1D system composed of large and small particles (graupel, hailstones or soft hail and ice crystals) and atmospheric ions has been investigated. For each collision between particles, charge transfer occurs. The amount of charge per collision and collision rate are random processes. Mutual forcing of such random events forms a charge spectra. The equations for collision integrals have been derived. On the basis of equations for distribution functions hydrodynamics approach has been proved. Also coagulation processes of crystals and graupel have been taken into consideration. The mean value and dispersion for stationary Gaussian spectra have been deduced. It has been shown that total stationary dispersion is much larger than the dispersion of charge per one collision.

INTRODUCTION and PROBLEM STATEMENT

A problem of charge distribution and charge fluctuations on cloud particles is principal for cloud electrification analysis, particularly for the grounding of electrohydrodynamic equations of charging widely used for charging process description. A multi-flow 1D system composed of cloud particles with bimodal size distribution (large graupel and small ice crystals with a fixed size) and air ions, under external electric (**E**) and gravity (**g**) fields have been examined. Charges of ice crystals and graupel vary in time and space. Due to the difference of mean velocities, particles of different sorts collide with each other. For each collision between ice and hail particle, an ice crystal acquires the charge δq while a hail particle acquires the opposite charge -δq. Here

$$\delta q = \zeta - \alpha q + \beta Q, \tag{1}$$

ζ is the random quantity with a given mean value δq_0 and the dispersion D_ζ; q and Q are the particles charges before an interaction, α , β are the constants depending on particle capacitance, conductivity and time of contact. The chain of colliding events is considered as the Markov process [1]. Magnitude and sign of δq_0 are determined by many parameters: external electric field (so called inductive charging), time contact, air temperature and humidity, particle's conductivity and elasticity, their melting (for ice particles), differential velocity **U**, see [2-5]. Dispersion of ζ is the result of three main reasons: random contact angle, random shape of ice crystal and hailstone surfaces and gas flow fluctuations. The distribution of ζ is supposed to be normal.

Let $f(q)\Delta q$ and $F(Q)\Delta Q$ are numbers of small and large particles at the intervals Δq and ΔQ. Then equations for distribution functions f and F can be written as:

$$\frac{df}{dt} = \{\partial_t f\}_i + \{\partial_t f\}_c, \ \frac{dF}{dt} = \{\partial_t F\}_i + \{\partial_t F\}_c. \tag{2}$$

The left part of equations contains sum of ionic and dust collision integrals. Unless otherwise stated f and F are normalized to 1: $\int f dq = 1$, $\int F dQ = 1$.

KINETIC EQUATIONS of CHARGING

To deduce ionic collision integral let us consider an area of f as it has been shown in Fig.1.a). Probabilities of transition are

$$P_+(x-1) = \gamma_+(x-1)\Delta t, \ P_+(x) = \gamma_+(x)\Delta t, \ P_-(x+1) = \gamma_-(x+1)\Delta t, \ P_-(x) = \gamma_-(x)\Delta t, \tag{3}$$

where $x = q/e$, $\gamma_\pm$ are effective ion charging frequencies. So, the f increment is

$$\Delta f(x) = P_+(x-1)f(x-1) + P_-(x+1)f(x+1) - (P_+(x) + P_-(x))f(x). \tag{4}$$

Substituting it to (4) produces a well-known result:

$$\{\partial_t f\}_i = \gamma_+(x-1)f(x-1) + \gamma_-(x+1)f(x+1) - f(x)(\gamma_+(x) + \gamma_-(x)). \tag{5}$$

It can be moved to partial derivative equation of the infinite order. Using Fokker-Plank assumption, the equation ((5)) turns to the second order one:

$$\{\partial_t f\}_i \approx f(\gamma'_- - \gamma'_+) + f'(\gamma_- - \gamma_+ + \gamma'_+ - \gamma'_-) + f''(\gamma_+ + \gamma_- + \gamma'_- - \gamma'_+); \tag{6}$$

when we suppose $\gamma_\pm(x)$ rather smooth function and leave only the first derivative on it. The other steps depend on structure of $\gamma_\pm(x)$. For small electric field it is determined by ionic diffusion, see [6],[7]:

$$\gamma_\pm = \frac{\nu_{i\pm}}{2}\frac{\tilde{x}}{sh(\tilde{x}/2)}\exp(\mp\tilde{x}/2), \tilde{x} = \frac{x}{x_T}, \tag{7}$$

where $\nu_{i\pm} = 4\pi\sigma_{i\pm}$, $\sigma_{i\pm}$ is ion conductivity, $x_T = RT_i/e^2$ is the effective thermal charge (in electron charges, e), R is the mean cloud particle radius, T_i is ions temperature in energy units. Further it will be assumed that charge per one collision is relatively large, $\delta q_0/e \gg x_T$, and all calculation will be performed at the area $x \gg x_T$. It simplifies collision integral calculation and the last can be written as:

$$\{\partial_t f\}_i \approx \nu_{i\pm}\partial_x(f(x)x) \tag{8}$$

Such approach gives Dirac-type stationary distribution $f(x) \sim \delta(x)$ like in a case $T_i \to 0$. Positive sign in (8) is for negatively charged particle, negative for positive one.

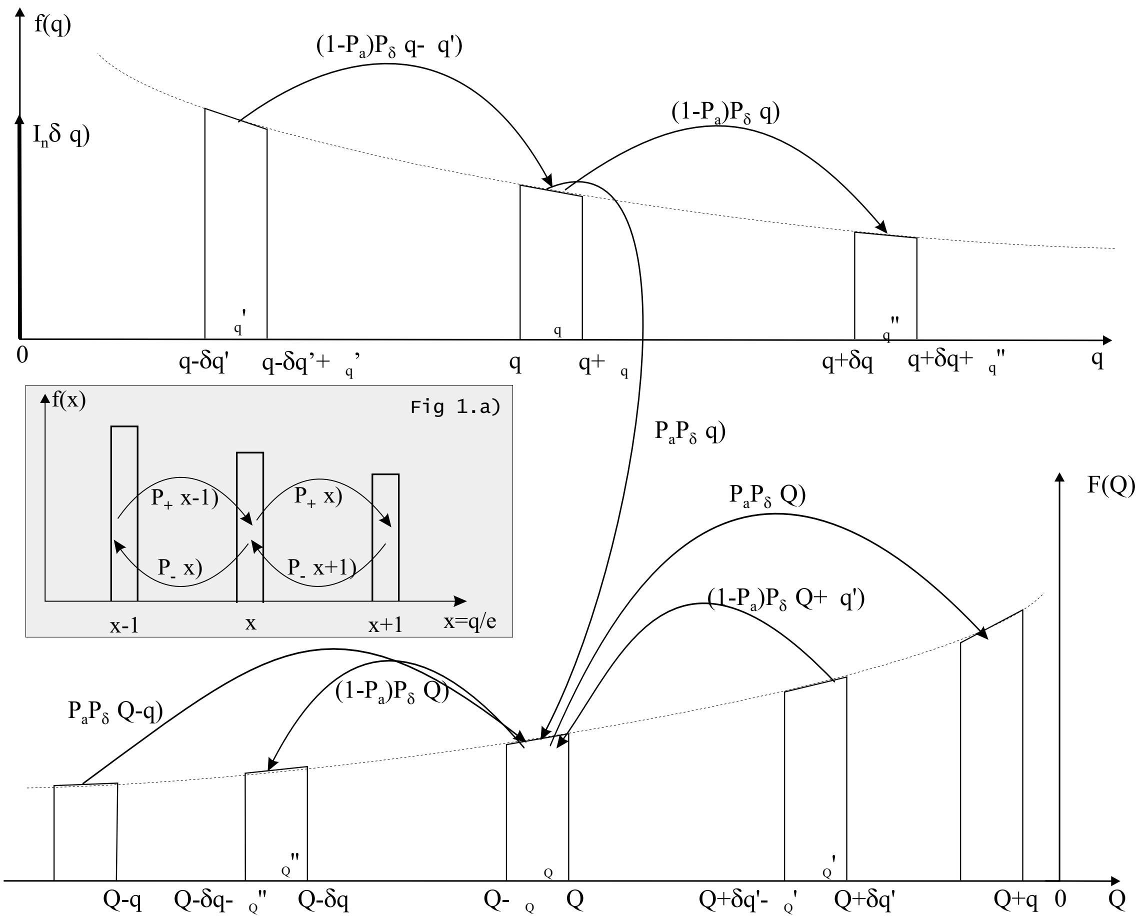

Figure 1: Probabilities of charge transitions due to particle-ions interaction Fig 1.a). Approximate diagram of ice-graupel interactions.

For particle collision integral, probabilities of transitions for a time Δt are expressed out through collision frequencies:

$$P_\delta(q - \delta q') = \nu_q(q - \delta q')\Delta t,\ P_\delta(q) = \nu_q(q)\Delta t, \tag{9}$$

where $\delta q'$ is the charge acquired by ice crystal for one collision with the graupel. It is important that $\delta q' \neq \delta q$, since q and Q need to be taken before collision event. Substituting $q - \delta q'$ against q and changing δq to $\delta q'$ gives:

$$\delta q' = \frac{\delta q}{1-\alpha} = \frac{\zeta - \alpha q + \beta Q}{1-\alpha}. \tag{10}$$

Also it is necessary to take into account that $\Delta_q \neq \Delta'_q$. The following definition is valid for $\delta q''$ as it is seen from Fig. 1:

$$\delta q'' = \delta q' + \Delta_q - \Delta'_q = \frac{\delta q}{1-\alpha} + \Delta_q - \Delta'_q, \tag{11}$$

but on the other hand

$$\delta q'' = \zeta - \alpha(q - \delta q' + \Delta'_q) + \beta Q = \frac{\delta q}{1-\alpha} - \alpha\Delta'_q. \tag{12}$$

Substituting it to (11) determines Δ'_q:

$$\Delta'_q = \frac{\Delta_q}{1-\alpha}. \tag{13}$$

Changing of particles number, $df(q)\Delta_q$, for the time Δt is determined by equation:

$$df(q)\Delta_q = \Delta t\{\Delta'_q \int\int \nu_q(\frac{q-\zeta-\beta Q}{1-\alpha})f(\frac{q-\zeta-\beta Q}{1-\alpha})F(Q)g(\zeta)dQd\zeta - \Delta_q\nu_q(q)f(q)\}, \tag{14}$$

where $g(\zeta)$ is the distribution function for ζ. Here it is taken into consideration that

$$q - \delta q = q - \frac{\zeta - \alpha q + \beta Q}{1-\alpha} = \frac{q-\zeta-\beta Q}{1-\alpha}.$$

Integration in (14) is performed over all range of $g(\zeta)$, $F(Q)$. The same procedure of deduction (14) can be performed for large graupel to obtain equation for $F(Q)$. So the full system of equations with ionic integrals and neglecting coagulation be as follows:

$$\frac{df}{dt} = \nu_i\frac{\partial(fq)}{\partial q} + \frac{1}{1-\alpha}\int\int \nu_q\left(\frac{q-\zeta-\beta Q}{1-\alpha}\right) f\left(\frac{q-\zeta-\beta Q}{1-\alpha}\right)F(Q)g(\zeta)dQd\zeta - \nu_q(q)f(q); \tag{15}$$

$$\frac{dF}{dt} = \nu_i\frac{\partial(FQ)}{\partial Q} + \frac{1}{1-\beta}\int\int \nu_Q\left(\frac{Q+\zeta-\alpha q}{1-\beta}\right) F\left(\frac{Q+\zeta-\alpha q}{1-\beta}\right)f(q)g(\zeta)dqd\zeta - \nu_Q(Q)F(Q) \tag{16}$$

where it is set $\nu_{i+} = \nu_{i-} = \nu_i$ otherwise, it is needed to split each spectra for positive and negative charges.

In practical manner,it is important to analyze $F(Q)$ and $f(q)$ when ice crystal has nonzero probability, P_a, to adhere with graupel. Here and after P_a is a constant value, although there are no principal difficulties in extending consideration to a more general case: $P_a = P_a(q, Q, d, D)$.

Let us assume that a source for neutral ice crystals exists, $I_n\delta(q)$, see Fig.1. Then ice concentration, n, can reach a stationary value n_0. As it is seen in Fig.1, change of f(q) at the interval Δ_q is a result of ice crystals income from the interval $q - \delta q' + \Delta'_q$ (but multiplying on $1-P_a$), particles outcome to interval $q + \delta q$ (with probability $(1-P_a)P_\delta$) and their adhesion with probability P_aP_δ. Also we need to integrate the probability of income over all charge spectra of the graupel. It gives us an equation for f_r:

$$\frac{df_r}{dt} = \nu_i\frac{\partial(f_rq)}{\partial q} + SU\frac{1-P_a}{1-\alpha}\int\int f_r\left(\frac{q-\zeta-\beta Q}{1-\alpha}\right)F_r(Q)g(\zeta)dQd\zeta - \nu_qf_r, \tag{17}$$

where S, U are the collision cross section and difference of cloud particles velocities, f_r, F_r are distribution functions with a new norm:

$$\int f_rdq = n, \int F_rdQ = N, \int f_rqdq = \rho_q, \int F_rQdQ = \rho_Q \tag{18}$$

For F_r we need to consider transitions from $Q - q$ (with probability P_a) and integrate over all ice crystals spectra:

$$\frac{dF_r}{dt} = \nu_i\frac{\partial(F_rQ)}{\partial Q} + SU\frac{1-P_a}{1-\beta}\int\int F_r\left(\frac{Q+\zeta-\alpha q}{1-\beta}\right)f_r(q)g(\zeta)dqd\zeta + \tag{19}$$

$$+SUP_a\int F_r(Q-q)f_r(q)dq - SUn(t)F_r.$$

HYDRODYNAMIC EQUATIONS of CHARGING

Direct integration (15) - (16) over the spectra gives a conservation law for cloud particles number: $n = const$, $N = const$. To get hydrodynamic (HD) equations, we just need to make standard procedure to system (15) - (16): $\int q\cdot(15)dq$ and $\int Q\cdot(16)dQ$. It yields a system:

$$\frac{dq_0}{d\tau} = -(\alpha k_1 + 1)q_0 + k_1\delta q_0 + k_1\beta Q_0; \quad \frac{dQ_0}{d\tau} = -(\beta k_2 + 1)Q_0 - k_2\delta q_0 + k_2\alpha q_0, \tag{20}$$

where $\tau = t\nu_i$, $k_1 = \nu_q/\nu_i$, $k_2 = \nu_Q/\nu_i$, $k_0 = 1 + k_1 + k_2$. The equivalent system for charge density is:

$$\frac{d\rho_q}{dt} = J - \rho_q(\nu_i + \alpha\nu_q) + \beta\nu_Q\rho_Q; \quad (21)$$

$$\frac{d\rho_Q}{dt} = -J - \rho_Q(\nu_i + \beta\nu_Q) + \alpha\nu_q\rho_q, \quad (22)$$

where $\mathbf{J} = \delta\mathbf{q_0SUNn}$ is the charging current. The equations above have been used in [8] for instability analyses of 1D quasi-electrostatic waves in 4-flow system with external electric field. Stationary charge densities can be expressed as $\rho_q = -\rho_Q = J/\nu_R$ that is a consequence of equal ion conductivities: $\sigma_{i+} = \sigma_{i-}$. Stationary charges are simply:

$$q_0 = \delta q_0 \frac{\nu_q}{\nu_R}, \; Q_0 = \delta q_0 \frac{\nu_Q}{\nu_R}, \quad (23)$$

where $\nu_R = \nu_i + \alpha\nu_q + \beta\nu_Q$ is the characteristic dissipation time.

When viewed coagulation, HD equations have been derived from the system (17)-(19). The conservation principle yields: $N = const$, $dn/dt = I_n - P_a\nu_a n$. It is evident that the rate of concentration set is determined by time $\tau_n = 1/(P_a\nu_q)$, stationary state, n_0, is simply $\tau_n I_n$. The equations for ρ_q, ρ_Q are as follows:

$$\frac{d\rho_q}{dt} = J(1-P_a) - \rho_q(\nu_i + \nu_q\alpha(1-P_a)) + \rho_Q\nu_Q\beta(1-P_a) - \rho_q\nu_q P_a; \quad (24)$$

$$\frac{d\rho_Q}{dt} = -J(1-P_a) - \rho_Q(\nu_i + \nu_Q\beta(1-P_a)) + \rho_q\nu_q\alpha(1-P_a) + \rho_q\nu_q P_a, \quad (25)$$

where $\nu_Q = \nu_Q(t)$, since $\nu_Q = SUn(t)$. The full stationary charge densities are:

$$\rho_q = -\rho_Q = \frac{J(1-P_a)}{\nu_R(1-P_a) + (\nu_q + \nu_i)P_a}. \quad (26)$$

APPROXIMATE GAUSSIAN SOLUTIONS

The simplest case is stationary spectra which looks like a Gaussian one. To obtain dispersions (mean values, q_0, Q_0 have been derived above) we need to multiply equations (15), (16) to q^2 and Q^2 respectively. After integrations over all spectra and simple transformations we obtain:

$$D_q = k_1(1+\beta k_2)\frac{D_\zeta + \frac{\delta q_0^2}{k_0^2}}{k_0 + 2\alpha\beta k_1 k_2 - \alpha^2 k_1(1+\beta k_2) - \beta^2 k_2(1+\alpha k_1)}, \;\; D_Q = D_q \frac{k_2}{k_1}\frac{1+\alpha k_1}{1+\beta k_2}. \quad (27)$$

References

[1] W. Feller, An introduction to probability theory and its applications, 3d edition, J.Wiley & Sons, 1970.

[2] M.B.Baker, J.Latham, K.Norville, A Numerical Study of Thunderstorm Electrification: Model Development and Case Study, *J.Geophys.Res.*, 96, 7463-7481, 1991.

[3] W.D.Keith, C.P.R.Sounders, Charge transfer during multiple large ice-crystal interactions with a riming target, *J. Geophys. Res.*, 94, p.13103-13106, 1989.

[4] M.B.Baker, J.G.Dash, Charge Transfer in Thunderstorms and the Surface Melting of Ice.*J.Cryst.Growth*, 97, 770-776, 1989.

[5] K.V.Berd, H.T.Och, Charging Mechanisms in Clouds and Thunderstorms, in *Earth's Electrical Environment*, pp.114-130 , edited by P.Krider and R.J.Roble, National Academy Press, Washington, D.C., 1986.

[6] G.F.Drukarev, Charges of raindrops, *Izv USSR Acad. Sc.*, v.8, N 6, pp. 330-336, 1946 (in Russian).

[7] N.A. Fuchs, To charge magnitudes on atmospheric aero colloids, *Izv. Academy of Science USSR, geography and geophysics series*, v.11, N4, pp.341-347, 1947 (in Russian).

[8] E.A. Mareev, A.E. Sorokin, V. Yu. Trahktengerts, Effects of Collective Charging in Multiflow Aerosol Plasma, Plasma Physics Reports, v. 25, N. 3, pp. 261-272, 1999.