PROOF OF CLOUD INSTABILITY WITH RESPECT TO THE FORMATION OF SEVERAL HORIZONTAL SPACE CHARGE LAYERS

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ABSTRACT: The origin of the multiple horizontal charge layers observed in the trailing stratiform region of mesoscale convective systems is explained for the first time on the basis of energy considerations and stability calculations. The latter indicate the presence of a cloud-instability with respect to formation of horizontal polarization layers and associated space charge layers. The author's polarization catastrophe theory of atmospheric electricity has inspired these calculations. This is consistent with the author's earlier derivation of charges in thunderclouds, in growing stratus clouds, in dissipating stratus clouds, an in the C-ring of the planet Saturn.

INTRODUCTION

For the first time we present rigorous proof of the instability of clouds containing small ice crystals, or sufficiently small H_2O aggregates in general, to the spontaneous formation of several horizontal polarization layers and space charge layers. Our investigation was inspired by the balloon observations (Fig. 1) of many horizontal space charge layers in

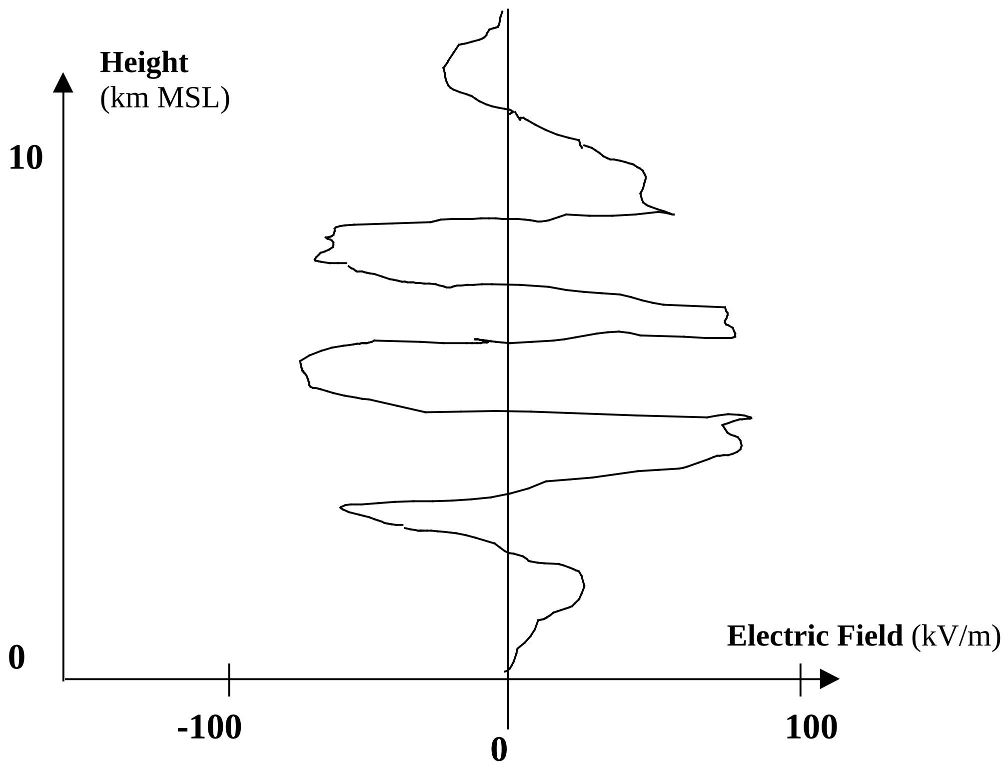

Fig. 1: Horizontal space charge layers found in MCS stratiform regions, typical curve.

conditions of the trailing region of Mesoscale Convective Systems (MCS). The method used by us was suggested by the success of the author's polarization catastrophe approach to cloud and thundercloud electrification, which later proved to be applicable also in the rings of Saturn.

Our proof consists two parts. In the first part we start from the dipole moment of a single ice crystallite, and then derive the expression of the energy of a cloud of such crystallites. We then prove that the energy of a uniform cloud with sufficient ice crystallite concentration is lowered by the spontaneous formation of layers of polarization and polarization charge, partially compensated by real masking charges.

The energy is given by the integral from the cloud base to clod top, of $n/\alpha - n^2/3\varepsilon_0$, where α is a polarizability constant of the individual ice crystallite, and n =n(z) is the initially constant concentration of ice crystallites as a function of height z. We denoted with ε_0 the vacuum permittivity. The author's well-known simple polarization catastrophe criterion requiring a sufficient concentration of crystallites is usually satisfied in most clouds, and this is used in the derivation.

In the second part, we show that an arbitrarily small sinusoidal displacement of the cloud results in a perturbation of the concentration of crystallites and will grow exponentially. Furthermore, starting from a cloud-lagrangian incorporating among other contributions also the energy contributions derived before, we show that an arbitrarily small sinusoidal perturbation of the cloud concentration will grow exponentially, indicating the presence of the instability that had to be proven.

PERTURBATION LOWERS THE ENERGY OF A CLOUD OF ICE CRYSTALLITES

Let n be the concentration of polarizable ice crystallites, each of them in the local field

$$E_{loc} = E_o + P/3\varepsilon_o, \qquad (1)$$

where E_1 is the macroscopic electric field in the cloud, and P is the cloud polarization, defined as the dipole moment per m^3. Considering each crystallite as comprising two charges +q and -q separated by the dipole length x, we obtain the average contribution of the dipole moment dipole moment of an ice crystallite to the polarization vector

$$p = qx = \alpha E_{loc} = \alpha F/q, \qquad (2)$$

where F is the force opposing the separation of the charges in the crystallite. The work u' done in creating the dipoles in the ice crystals per unit cloud volume is

$$u' = n\int F dx = (nq^2/\alpha)\int x dx = nq^2x^2/2\alpha = np^2/2\alpha = P^2/2n\alpha. \qquad (3)$$

This increases the energy of the cloud, just like the energy of a set of extended springs. Here P =np. On the other hand, the cloud loses energy due to the polarization of the crystallites,

$$u'' = \int E_{loc} dP = \int (E_o + P/3\varepsilon_o) = P^2/6\varepsilon_o + \int E_o dP. \qquad (4)$$

The energy of the cloud is thus

$$u = u' - u'' = (p^2/2)[n/\alpha - n^2/3\varepsilon_o]. \qquad (5)$$

The electric field E_o was assumed to be largely compensated by the masking charges of ions attracted to the regions where the divergence of P differs from zero, and will be neglected. A more detailed analysis shows that indeed, for processes that are not too fast, it is correct to assume E_o to be small, while E_{loc} may not be small. Here we may assume that the cloud polarization P in Eq. (4) is saturation polarization P =np, with p being the total dipole moment of each crystallite. Indeed, the author's polarization catastrophe criterion is (Handel, 1984, 1985; Handel and James, 1983)

$$\nu^2 n > 2.5 \cdot 10^{21}\ \mathrm{cm}^{-3}, \qquad (6)$$

where ν is the number of water molecules in each crystallite and n is their number per unit cloud volume. This criterion is satisfied in most clouds. However, our derivation is more general.

Let us apply a displacement

$$\xi(z, t) = \xi_o(t) \sin kz \qquad (7)$$

with an arbitrarily small amplitude $\xi_o(t)$ to the whole cloud that was assumed to be initially homogeneous, with $n(z)=n_o$ assumed to be initially constant over the whole cloud, from z=0 at the cloud base, to z =h at cloud top. Then, from the equation of continuity, we obtain in first order the concentration perturbation

$$\delta n = -(d/dz)(\xi n_o), \text{ i.e, } n(z,t) = n_o - n_o \xi_o(t)\, k \cos kz. \qquad (8)$$

Substituting into the expression of the relevant energy of the cloud

$$U = (p^2/2)\int [n/\alpha - n^2/3\varepsilon_o] \qquad (9)$$

we obtain

$$\delta U = (p/2)\int \left[-(n_o\xi_o/\alpha)\, k \cos kz + (2\xi_o/3\varepsilon_o)\, k \cos kz - (\xi_o^2 k^2 n_o^2/3\varepsilon_o) \cos^2 kz \right] dz \qquad (10)$$

Averaging over z, we obtain, at least for $k=2\pi r/h$ with integer r, $\langle \cos kz \rangle=0$, and $\langle \cos^2 kz \rangle.=1/2$. Considering first $\xi_o(t)$ constant, we obtain a negative result

$$\langle \delta U \rangle = -(p\xi_o k n_o/2)^2/3\varepsilon_o \qquad (11)$$

Therefore, the creation of regions of enhanced polarization sandwiching regions of reduced polarization, lowers the cloud energy. This concludes the elementary proof.

We now find the time dependence of $\xi_o(t)$, using the lagrangian approach.

LAGRANGIAN APPROACH

The hamiltonian and the lagrangian are given by

$$H = \int \mathcal{H} dz = \int \Pi^2 dz/2nm + \iint \rho_p(z)\rho_p(z')dzdz'/2|z-z'| + (p^2/2)\int[n/\alpha - n^2/3\varepsilon_o]dz; \quad (12)$$

$$L = \int \mathcal{L} dz = \int nm(d\xi/dt)^2 dz/2 - \iint \rho_p(z)\rho_p(z')dzdz'/2|z-z'| - (p^2/2)\int[n/\alpha - n^2/3\varepsilon_o]dz, \quad (13)$$

where $\mathcal{H}$ and L are the corresponding hamiltonian and lagrangian densities, $\Pi = n(z,t)md\xi/dt$ is the momentum density, m the mass of a crystallite, $\rho_p(z)$ is the polarization charge. The latter is again considered to be largely compensated by the real charge of congregating ions, and is therefore neglected. We therefore neglect the central term. Substituting n from Eqs. (8) and (11) and averaging,, we obtain

$$<\mathcal{L}> = <(m/2)n_o[1-\xi_0(t)\, k \cos kz](d\xi_0/dt)^2 \sin^2 kz -(p^2/2)[n/\alpha - n^2/3\varepsilon_o]> -(n_0{}^2p^2/2)[n/\alpha - n^2/3\varepsilon_o]$$

$$-(p\xi_o k n_o/2)^2/3\varepsilon_o. \quad (14)$$

Expressing sinkz through cos kz, we notice that odd powers of cos kz yield zero in average. The motion is obtained from

$$<(d/dt)[\partial\mathcal{L} / (d\xi_0(t)/dt)] - \partial\mathcal{L} /\partial t> = 0, \quad (15)$$

i.e,

$$d^2\xi/d\xi^2 - (pkn_o)^2\xi_o/3m\varepsilon_o = 0 \quad (16)$$

Thus,

$$\xi_0(t) = \xi_{00} \exp(pn_o kt/3^{1/2}m^{1/2}\varepsilon_o{}^{1/2}). \quad (17)$$

This proves the presence of the instability. The negative exponential is eliminated by the energy integral obtained by amplifying Eq. (16) by $d\xi_o/dt$ and noticing the resulting increase of the energy of the perturbation in time.

CONCLUSIONS

In most quiescent clouds we expect a stratification with alternating positive and negative layers, due to an inherent instability. The pattern may be triggered by small initial charge and polarization fluctuations.

REFERENCES

P.H. Handel: "Polarization Catastrophe Theory of Cloud Electricity, a New Mechanism for Thunderstorm Electrification", J. Geophysical Research 90, 5857-5863 (1985; first submitted in 1980).

P.H. Handel: "New Approach to Cloud Electrification and Lightning", Proc. VII. International Atmospheric Electricity Conference, Albany NY, June 1984, SUNYA Press, p.190-195.

P.H. Handel and P.B. James: "Polarization Catastrophe Model of Static Electrification and Spokes in the B-Ring of Saturn", Geophys. Res. Lett. 10, 1-4 (1983).