ABSTRACT: Return stroke current pulse can propagate as fast as at a significant fraction of the speed of light c. Such a fast-moving pulse is expected to radiate differently than the conventional RF emitters. In this report, we will carry out a theoretic analysis for the high-speed effect on the radiation beam pattern. The theoretic analysis is based on relativistic considerations, and it differs from previously reported, classical electrodynamic-based analyses. We then investigate the FORTE observations of return strokes. FORTE obtained VHF radiation signals from tens of thousands of return strokes over the contiguous United States in the summers of 1998 and 1999. About 10% of the return strokes were initiated with a very narrow (< 100 ns), intense, and highly linearly polarized radiation pulse. Statistic examinations of these pulses indicate that they form a beam pattern that tilts upward, in agreement with the model predication for a propagating current pulse at speed of ~0.6c.

INTRODUCTION

Electromagnetic radiation produced by a fast-propagating current pulse will defer from a dipole radiation, especially in the aspect of beam pattern. *Le Vine and Willett* [1992] studied these effects with a conventional EM approach in which careful quasi-static time-space geometries were examined for the effective pileup or corruption of the radiation field. They have further tested their theoretic models against ground-based, multiple-station electric field change observations. *Thottappillil et al.*, [1998] also investigated the propagation effects based on conventional geometric approaches, and came up with similar results.

The FORTE satellite provides, for the first time, systematic VHF lightning observations from the up-half space. This unique capability would allow us to examine more directly the VHF radiation beam pattern of the observed return strokes.

In this paper, we will revisit the propagation effects on the radiation field with a different, more concise approach. We will also analyze the FORTE observations of return strokes to reveal the possible beam patterns, and will compare the observed results with the theoretic models.

THEORETIC ANALYSIS

Figure 1 shows the problem under consideration, in which a current pulse propagates upward along the lightning channel and its produced radiation field is measured at point P(x, y, z). The instantaneous vector potential due to the current segment (dz') is

\[ d\tilde{A}(t, \tilde{r}) = \frac{\mu_0}{4\pi} \frac{k' z'}{r(z')} dz', \quad t' = t - \frac{r(x, y, z, z')}{c} \]

Here, dz' is assumed to move along with the current pulse at speed v. The retarded time t' is an implicit function of (x, y, z), so that any differential operation on (x, y, z) in the stationary frame would need to operate on t' too. This leads to

\[ d\tilde{B} = \nabla \times (d\tilde{A}) = \nabla \times (d\tilde{A}) \big|_{t=const} + \nabla t' \times \frac{\partial(d\tilde{A})}{\partial t'} \]

In the (x, y, z) coordinates, t is uniform and \( \nabla t \neq 0 \), but viewing from stationary P, \( \nabla t' \) may not be zero, due to the nonlinear compressions of the space and time, so that from Equation 1 we have

\[ \nabla t' = -\frac{1}{c} \nabla r = -\frac{1}{c} \nabla r \big|_{t=const} - \frac{1}{c} \frac{\partial r(t')}{\partial t'} \nabla t' = -\frac{\tilde{r}}{cr} + \nabla t' \frac{(z - z') \frac{\partial z'}{\partial t'}}{\sqrt{x^2 + y^2 + (z - z')^2}} = -\frac{\tilde{r}}{cr} + \frac{\nabla t' \cdot \tilde{r}}{c} \]

Rearranging the above equation, we have

\[ \nabla t' = -\frac{\tilde{r}}{c(r - \tilde{v} \cdot \tilde{r} / c)} = -\frac{\tilde{r}}{c(1 - \tilde{v} \cdot \tilde{r} / c)} \]
Since \( d\hat{A} \propto 1/r \) so that \( \nabla \times (d\hat{A})/r_{\text{const}} \propto 1/r^2 \). For radiation field, this term can be neglected as compared to the \( 1/r \) terms. Put these together back into Equation 2, we have

\[
d\vec{B} = -\frac{\hat{r}}{c(1-\hat{v} \cdot \hat{r}/c)} \times \mu_0 \frac{1}{4\pi r} \frac{1}{\sin \theta} \frac{\partial \hat{i}(\hat{z},t')}{\partial t'} dz'd\hat{\alpha} = \mu_0 \frac{1}{4\pi r} \frac{1}{(1-\hat{v} \cdot \hat{r}/c)} \frac{\sin \theta}{\partial t'} \frac{\partial \hat{i}(\hat{z},t')}{\partial t'} \frac{dz'd\hat{\alpha}}{1-\hat{v} \cdot \hat{r}/c}
\]  

(5)

For plane EM wave, which is justified for distant observation, we have

\[
d\vec{E} = -c\hat{r} \times \vec{B} = \frac{1}{4\pi c^2 r} \frac{\sin \theta}{(1-\hat{v} \cdot \hat{r}/c)} \frac{1}{\sin \theta} \frac{\partial \hat{i}(\hat{z},t')}{\partial t'} dz'd\hat{\alpha}
\]

(6)

Equation 6 shows that the observed radiation depends on the rate of current change at the source at time \( t' \). The corresponding radiation beam pattern depends not only on the direction but also on the speed of the propagating pulse. For a non-propagating pulse \( (v = 0) \), the beam will become a dipole pattern. Equation 6 is a general result for a small scale \( (dz') \) discharge, and can be readily tested against measurement of impulsive VHF radiation. In addition, the propagation does not have to be vertical but can be in any direction. It is in the case of return stroke that the propagation is assumed vertical.

For an extended discharge, the radiation field can be obtained by proper integration of Equation 6. Assuming the discharge starts at \((z'=0, t'=0)\), at \( t \) in the stationary frame \((x, y, z, t)\), the discharge will extend up to \( L' = vt' \), and the integration can be expressed as

\[
\vec{E} = \frac{1}{4\pi c^2} \frac{\sin \theta \hat{\alpha}}{r} \int_0^r \frac{\sin \theta}{(1-\hat{v} \cdot \hat{r}/c)} \frac{\partial \hat{i}(\hat{z},t')}{\partial t'} dz' = \frac{1}{4\pi c^2} \frac{\sin \theta \hat{\alpha}}{r} \int_0^r \frac{\sin \theta}{(1-\hat{v} \cdot \hat{r}/c)} \frac{\partial \hat{i}(\hat{z},t')}{\partial t'} dz' \hat{i}(vt', t') - \hat{i}(0,t')
\]

Here, we assumed \( L' \ll r \) so that \( \theta \) will remain the same along the channel. If the concept of the transmission line model is used, that is \( \hat{i}(vt', t') = \hat{i}(0, t') \), and \( \hat{i}(0, t') = 0 \), Equation 7 can be simplified as

\[
\vec{E} = -\frac{1}{4\pi c^2} \frac{\sin \theta \hat{\alpha}}{r} \hat{i}(0,t')
\]

(8)

If a perfect conducting ground is considered and if the discharge is vertical, under the condition of \( L' \cos \theta < \lambda/\pi \), we have

\[
\vec{E} = \frac{1}{4\pi c^2} \frac{\sin \theta}{r} \hat{i}(0, t') \hat{\alpha}
\]

(9)

Figure 2 compares beam patterns between a dipole and a traveling current pulse, in free space and on the ground. Propagating speed of 0.7c is used for the demonstrations. These results are in general agreement with what reported by Le Vine and Willett [1992] and Thottappillil et al. [1998], but are derived in a much more concise means.

FORTE VHF OBSERVATIONS

FORTE satellite is in a circular orbit of 800km. With its broadband VHF antennas and receivers, FORTE provides a unique capability of observing lightning radiation from the up-half space, and therefore offers an opportunity to investigate the VHF beam pattern directly. For a vertical ground stroke, the viewing angle referenced to the channel direction can be readily determined with known locations of the satellite and the stroke, as illustrated in Figure 3.

During the summers of 1998 and 1999 joint FORTE/NLDN campaigns were conducted, and 25721 coincidences were gathered. For this study, NLDN categorized the discharge types and provided the geolocations for the corresponding FORTE VHF signals. Of the coincidences, most are ground strokes of cloud-to-ground flashes. As being reported previously [Jacobson and Shao, 2002], ~10% of the ground strokes were accompanied by an extremely narrow (<100ns), intense, and highly linearly polarized VHF pulse. It was also
found that strokes with such a pulse are more likely to occur over the seawater than over the land. Radiation for the rest of the strokes appears to be continuous and unpolarized, as produced by normal return strokes.

Figure 4 shows the FORTE detected radiation intensity versus the zenith angle (viewing from the source, Figure 3) for all the coincident events (green) and for the subset narrow-pulse strokes (red). Not surprisingly, this plot appears to be very noisy and does not show any definite beam pattern, due to the nature of that different lightning strokes can have very different discharge intensities.

Instead of the radiation intensity, Figure 5 shows the number of FORTE detected events, normalized to a unit earth area at each zenith bin, versus the zenith angle. In the case of a single radiator and simultaneous all-sky view, this analysis is statistically equivalent to the detection probability of the radiator at different zenith angles. It is clear that the subset of the narrow pulse strokes displays a significantly different distribution as compared to that of the ensemble of all the events.

To reveal the possible beam pattern effect on these distributions, we assume that the occurrence of a discharge follows an exponential law respecting to its radiation intensity,

\[ \frac{n(x_E)}{N} = \frac{1}{a} e^{-x_E/a} \]  

in which \( x_E \) represents the radiation amplitude. Under considerations of beam pattern \( b(\theta) \), Equation 10 can be rewritten as

\[ \frac{n(x_E, \theta)}{N} = \frac{1}{ab(\theta)} e^{-\frac{x_E}{ab(\theta)}} \]  

At the other end, three more factors have to be considered: (1) the FORTE antenna beam pattern \( g(\theta) \), (2) the changing distance between the source and the satellite \( r(\theta) \), and (3) the FORTE receiver’s trigger threshold \( X_T \). Combining these factors with Equation 11, the probability of a signal to be detected by FORTE is

\[ \frac{\bar{N}(\theta)}{N} = \int_{X_T}^{\infty} \frac{r(\theta)}{ab(\theta)g(\theta)} \frac{X_Tr(\theta)}{ab(\theta)g(\theta)} dX_E = e^{-\frac{x_E}{ab(\theta)g(\theta)}} \]  

In this equation, \( a \) can be extracted from the measurement by a cross cut through the date points in Figure 4 at a chosen zenith angle, \( X_T \) can be retrieved form FORTE’s housekeeping records, \( g(\theta) \) is a known antenna parameter, and \( r(\theta) \) can be computed from the satellite’s orbit. Therefore, from a chosen beam pattern \( b(\theta) \), Equation 12 can be used to estimate the corresponding detection probability, or vice verse.

Figure 6 shows the simulated detection probability (red) by assuming an isotropic radiation pattern, i.e., \( b(\theta) = 1 \). For comparison, the actual distribution for the ensemble of all the events (black) is also shown in the figure. This comparison appears to suggest that all the 25721 FORTE/NLDN coincident events together resembles a near isotropic beam pattern in the up-half space. Due to the nature of the “single-shot” observation for each individual stroke and the nature of the statistic analysis, beam pattern for each stroke cannot be determined, and should not be assumed to have the same isotropic pattern. However, this all-event or “background” distribution can be used to examine the possible beam patterns for the subgroup of the narrow-pulse strokes.

If the background distribution can be indeed assumed isotropic, the ratio between the two curves in Figure 5 will reveal the corresponding beam pattern for the narrow-pulse strokes. This is show in Figure 7a, in which a conventional polar plot format is used. Figure 7b compares the inferred beam pattern with three different modeled patterns. The pattern is properly scaled to have the best comparison. Clearly, it does not agree with the dipole pattern. It agrees, to some extents, with the \( v = 0.6c \) propagating model that consists of the conducting ground. But best of all, it appears to be in good agreement with the free
space $v = 0.6c$ propagating model, indicating that the responsible current pulse may occurred somewhere above the ground. Previous studies (see the review by Uman [1987]) have shown that return strokes start with an attachment process that connects the normal downward leader with an upward leader at a point 10-50m above the ground. The intense, narrow pulse presented here is believed due to such an attachment process. At these heights, the ground reflection of the original pulse will be delayed by several tens to a few hundreds ns, depending on the viewing angle. Due to the impulsive nature of the pulse (<100ns), the reflected pulse will have little effect on the original pulse because of the time delay. In addition, the reflected pulse will appear weaker in amplitude due to the forwardly enhanced radiation patterns.

The inferred beam below $20^\circ$ elevation (elevation = $90^\circ$ - zenith) appears not to agree with any of the models. This could be caused by several different factors. First, at the low elevation angles, signal dispersion caused by the earth’s ionosphere (which is at the middle between the source and FORTE) will increase significantly and the reconstruction of the original signal waveform becomes more difficult. A dispersed signal will effectively have a broadened pulse width, and therefore decreases its chance being categorized as a narrow pulse. Secondly, more careful examination of Figure 6 indicates that the actual beam pattern for the background events may be “fatter” than the assumed isotropic pattern, as indicated by the higher detection probability at zenith angles near $70^\circ$ ($20^\circ$ elevation) over the isotropic model. This will effectively reduce the distribution ratio of the narrow-pulse to the background.

SUMMARY AND CONCLUSION
In this report, we reexamined the theoretic background for the radiation beam pattern due to a moving current pulse. Instead of using the classical electrodynamic approach, we implemented the relativistic concept, in which time-space in the moving coordinate frame is nonlinearly related to that of the stationary, observer’s frame. The derived result is the same as that reported before by other researchers with the conventional EM analysis, but the derivations are much simpler and concise.

We also studied the FORTE VHF observations of return strokes and compared the induced VHF beam patterns with the theoretic models. We found that the ensemble of all the FORTE detected stokes displays a near isotropic radiation pattern, whereas the subset of the strokes that are accompanied by a very narrow (<100ns), intense, and highly linearly polarized pulse shows an upward beam pattern. The upward beam pattern agrees with a free space current propagating model at $v = 0.6c$. This further indicates that the discharge processes responsible for these current pulses occurred somewhere above the ground (10s meters).

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REFERENCES