Homework 2	AT 741	Spring 2008

I. Computation of reflectivity-weighted axis ratios

(a) The table below contains data from a raindrop size distribution with four diameters (1-4 mm) and their associated concentrations in # of drops m^{-3}. The equilibrium axis ratio at each diameter can be approximated by the Purppacher and Beard (1970; referred to as PB 1970 in the table below) formula:

$$r = 1.03 - .062(D)$$

where D is the drop diameter in mm.

The reflectivity-weighted axis ratio of the complete drop size distribution can be computed by:

$$r_Z = \Sigma(\text{Z of each drop size bin * r for that drop size}) / \Sigma\ (\text{Z for each drop size bin})$$

The reflectivity contribution for each diameter bin comes from the basic Z defining equation involving the product of diameter raised to the sixth power and drop concentration. The required calculations may be aided by completing the following table:

D (diameter (mm))	N(D) (# m^{-3})	Z(mm^6m^{-3})	r(PB 1970)	Z*r
1	750	_____	_________	_______
2	188	_____	_________	_______
3	47	_____	_________	_______
4	5	_____	_________	_______
Summations:		_____		_______

What is the reflectivity-weighted mean axis ratio of this raindrop size distribution?

(b) Now assume that a component of small hail in the form of 12 mm diameter (essentially 0.5 inch), perfectly spherical, solid ice particles is added to the above raindrop population. The hail concentration is 1 stone per cubic meter. Re-calculate the Z-weighted mean axis ratio with the spherical hail contribution included. (i.e., Add a new line to the above table for the hail and repeat the Part I calculations. Note: the calculated Z value for the hail must be reduced by the ratio of the refractive index values for solid ice / water (i.e., 0.176 / 0.93).)

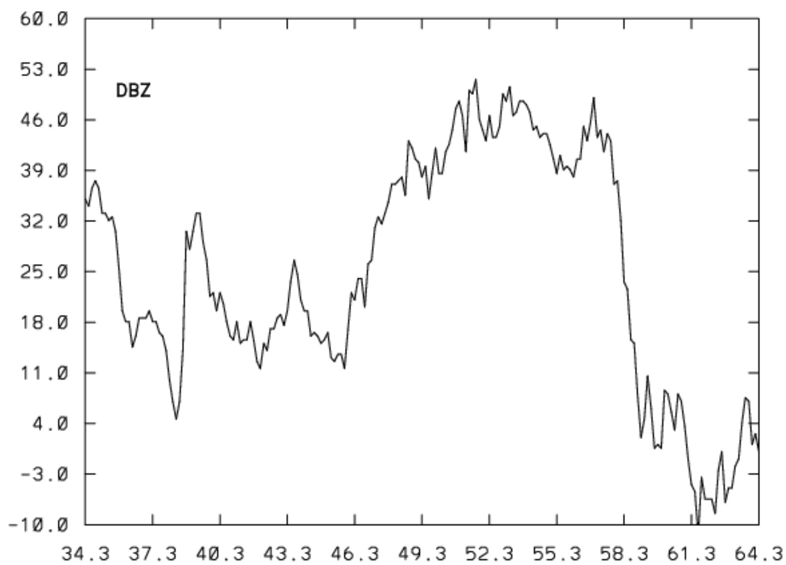
What is the reflectivity-weighted mean axis ratio for the rain + hail mixture?

II. Basic processing of a ray of ASCII format radar data

A file containing a portion of one ray of CSU-CHILL data collected on 7/16/2004 has been posted on the class website via the homework 2 link "section 2 data." This file contains columns of individual range gate data values with each row of values coming from a given range value. Here are the first few lines of the file:

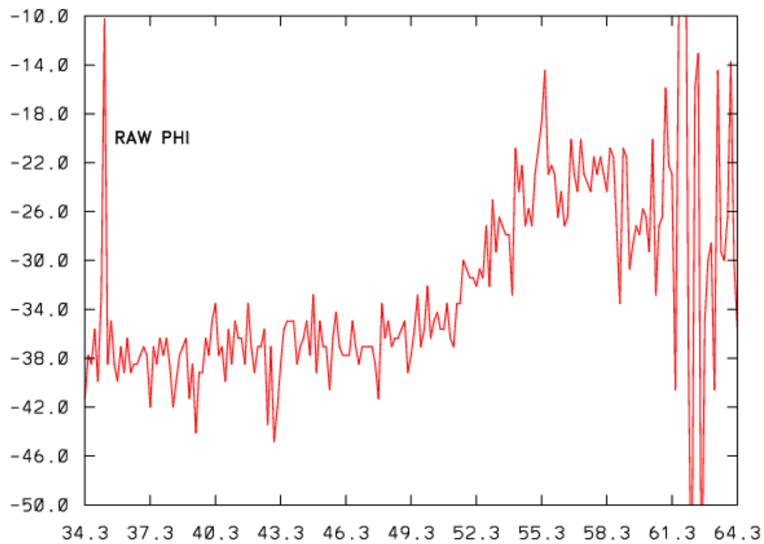
```
# azdeg= 216.34
# range(km)  dbz     vel    zdr   phidp    rhohv  ldr
  34.25    35.08   -2.08   0.59  -41.29   0.97  -25.59
  34.40    34.08   -1.87   0.92  -37.76   0.88  -23.52
  34.55    36.58   -2.71   0.78  -38.47   0.98  -24.46
(continues on to longer ranges)
```

For this problem, write a program using IDL, Matlab, etc. to ingest the data file. To make sure you have read the data in properly, make a simple X-Y plot of reflectivity vs. range in km. It should look like this (X axis is range in km, Y axis is reflectivity in dBZ):



The reflectivity profile includes a convective rain shaft in the general 46–58 km range interval.

A similar plot of the input differential propagation phase data values is shown below:

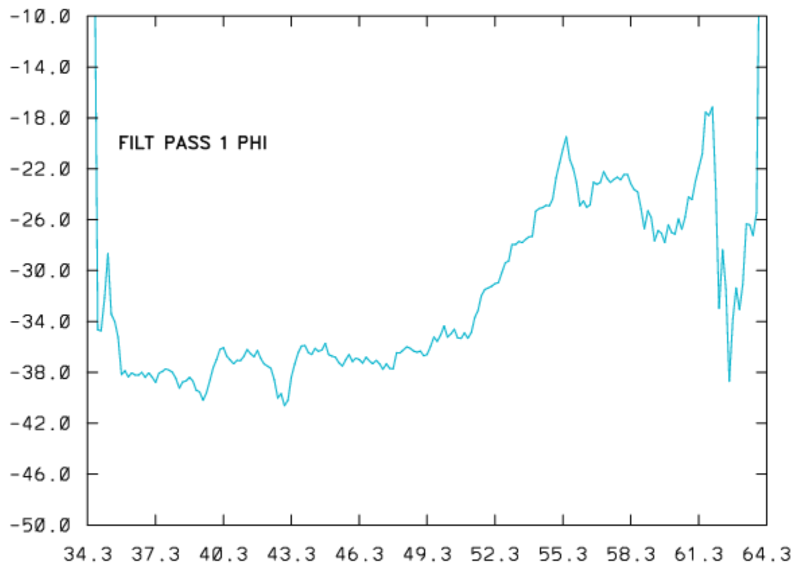


The high frequency variations in the phidp range profile (above) are generally filtered out before the estimation of the slope (K_{dp}) is undertaken. For the purposes of this problem, a simple 7 gate long "triangular weighting" filter will be applied to the phidp data values. The filtered phidp value at gate "n" is obtained by developing a weighted sum using the phidp values found in the neighboring +/- 3 gates:

.1 * phidp at gate n-3
.1 * phidp at gate n-2
.15 * phidp at gate n-1
.3 * phidp at the central gate where the filtered value will apply
.15 * phidp at gate n+1
.1 * phidp at gate n+2
.1 * phidp at gate n+3

(i.e., the weights sum up to 1.0 and are applied symmetrically around the central gate at which a range-filtered value is being developed).

Below is the phidp profile after one application of the 7 gate long filter:



To summarize, the data plotted above were generated by stepping along the basic input phi data array and summing up the central and neighboring +/- 3 gate phidp values using the symmetrical set of fractional weights shown on the last page.

Apply the range filter three successive times to the phidp data file (i.e, the second time around, the input will be the smoothed profile resulting from filter application number one, etc.) and note that the plot should change very little between passes 2 and 3. Then develop a K_{dp} estimate over each block of 10 range gates using both the original (unsmoothed) and final (filtered 3 times) phidp profiles (create a single K_{dp} estimate for each separate block of 10 phidp values: do not create an estimate at every range bin). The slope (K_{dp}) estimate is performed by making a least-squares linear fit to each sequence of 10 phidp gate values. A web reference for this type of estimate is the equation for b_1 given on the website: http://www.tufts.edu/~gdallal/slr.htm

Remember that the general meteorological convention is to use one way K_{dp} values; the least squares fit slopes need to be divided in half to get the desired expression of one way K_{dp} in ^{o} km^{-1}.

For this problem, provide the following:

- -a plot of your phidp series after the three applications of the 7 gate filter:
- -a plot of your K_{dp} values as a function of range based on both the original (unfiltered) phidp data and based on the phidp data that was subjected to the three filtering passes.
- -At approximately what reflectivity level (dBZ) do the K_{dp} values appear to become meaningful (i.e., they are not just random fluctuations)?
- -To what do you attribute the large raw phidp fluctuations at ranges beyond ~59 km?

III. Calculation of Z_{dr} from the Rayleigh-Gans dipole moments

An equation for the depolarization factor λ_z for oblate scatterer shapes is shown below:

$$\lambda_z(\text{oblate}) = \frac{1+f^2}{f^2}\left(1 - \frac{1}{f}\tan^{-1} f\right); \quad f^2 = \left(\frac{a}{b}\right)^2 - 1, \quad a/b \geq 1$$

In this formula, 'a' is the major (horizontal) radius of the particle and 'b' is the minor (vertical) radius (Note: Hydrometeor axis ratios are often presented with the opposite definition, which makes the axis ratio of oblate particles be less than one).

You will recall that Z_{dr} is based on the power ratio defined by $(S_{hh})^2 / (S_{vv})^2$. Under the Rayleigh–Gans approximation, equations for these two quantities are as follows (λ_z is the axis ratio-dependent term defined above):

$$S_{hh}(r, D) = \frac{k_0^2}{4\pi} \frac{V(\varepsilon_r - 1)}{\left[1 + \frac{1}{2}(1 - \lambda_z)(\varepsilon_r - 1)\right]}$$

$$S_{vv}(r, D) = \frac{k_0^2}{4\pi} \frac{V(\varepsilon_r - 1)}{[1 + \lambda_z(\varepsilon_r - 1)]}$$

When the S_{hh} / S_{vv} ratio is formed, it is apparent that the common particle volume (V), wavenumber (k_o) and 4π terms divide out. Thus the Z_{dr} for single particles can be calculated from the ratio of the remaining terms, which involve only the particle axis ratio (included in λ_z) and the relative permittivity dielectric factor (ε_r). Let ε_r for water be 80 and ε_r for ice be 3.0.

Calculate Z_{dr} (in dB) for the following cases:

a) Water drop with a=1.0 and b=0.7 according to the λ_z formula convention
b) Water drop with a=1.0 and b=0.5 according to the λ_z formula convention

c and d) Repeat the above two calculations for solid ice (.92 g cm^{-3} density):

Note: It is necessary to square the basic S_{hh}/S_{vv} ratio to convert to a linear power ratio; express this power ratio in dB to get the customary Z_{dr} units.