

Computer simulation of weather radar signals

Sutanay Choudhury , Nitin Bharadwaj
Department of Electrical & Computer Engineering
Colorado State University

Abstract:

Computer simulation is largely used in radar signal analysis. This present work treats the simulation of precipitation echoes in weather radar application, where two different approaches are used for simulating weather radar signals. These simulated signals offer great potential for estimating spectral moment and various polarimetric variables. One of the methods uses a time domain approach while the other uses the frequency domain approach. We show that the simulated signals reflect the desired properties. Both the simulation algorithms are based on a macroscopic model, i.e. a random processes with an assigned power spectrum.

1.Introduction :

Weather radar signals are a composite of echoes from a very large number of individual hydrometeors or from refractive index irregularities in clear air. The conventional single-polarized Doppler radar uses the measurement of radar reflectivity, radial velocity, and storm structure to infer some aspects of hydrometeor types and amounts. With the advent of dual-polarized radar techniques it is generally possible to achieve significantly higher accuracy. From simulation point of view, this implies the generation of two pseudorandom sequences with an assigned autocorrelation function and an assigned cross correlation.

Computer simulation of the precipitation echo is necessary to study signal processing and data extraction. By means of a signal simulator it is possible to evaluate the effectiveness of a signal processing algorithm in a controlled environment. The weather echoes contribute to produce a complex voltage $V=I+jQ$. A rain signal simulator shall provide in phase and in quadrature components of the channel in horizontal polarization, (I_H, Q_H) , and of the channel in the vertical polarization, (I_V, Q_V) with correct marginal and joint densities. As the rain echo is assumed to be a stationary Gaussian process it follows that it is completely characterized by its covariance matrix and it is sufficient to simulate correctly the autocovariance and mutual covariance between the H and the V.

The parameters involved in the selection of the generation method are N_p, Z, Z_{DR} , and the shape of the autocorrelation coefficient at lag $k, \rho(k), k>1$. In this project two methods are considered: the first, called the

FCG (Fast Convolution Generator) uses a fast convolution in the frequency domain, while the other creates the spectral components (Fourier harmonics) of the sequence.

2. Time series with an assigned polarization using FCG :

The weather echo, supposed stationary is characterized by the power spectral density given by

$$S_y(f) = \frac{1}{\sqrt{2\pi\sigma_f^2}} \exp\left[-\frac{(f-f_D)^2}{2\sigma_f^2}\right]$$

And a autocorrelation function as follows,

$$R(n) = E y^*(k) y(k+1), \quad n, k \text{ integers,}$$

Where f_D is the Doppler frequency and σ_f is the spectral width. The relationship between σ_f and velocity spread σ_v of the radial speed of hydrometeors is

$$\sigma_f = \frac{2\sigma_v}{\lambda},$$

Where λ is the radar wavelength.

The purpose of the generator is to generate a complex random signal having the properties (1) and (2). This is equivalent, as regards the spectral properties, to generate a time series with an autocorrelation coefficient equal to

$$\rho(n) = \exp\left[-2(\pi\sigma_f n T_S)^2\right] \cdot \exp[j2\pi f_D n T_S]$$

Where T_S is the pulse repetition period.

An FIR filter is used to implement the FCG. The underlying principle is based upon the "coloration" of a white spectrum by linear filtering. Complex Gaussian white noise ($S_x(f)=1$) is colored by an FIR filter. The output time series obtained from this filter in the frequency domain is

$$Y(f) = H(f) \cdot X(f)$$

Where $Y(f)$ and $H(f)$ are the DFTs of $y(n)$ and $x(n)$ respectively and $H(f)$ is the FIR filter. The power spectra of $y(n)$ is

$$S_y(f) = |H(f)|^2 \cdot S_x(f),$$

which implies that

$$S_y(f) = |H(f)|^2,$$

$S_y(f)$ is real and positive since it's the *Fourier transform of the autocorrelation function*. Hence the filter transfer function is obtained as given below

$$H(f) = \sqrt{S_y(f)}$$

Now, the simulation algorithm is as follows:

- (1).Generate unitary power “white” uncorrelated Gaussian samples.
- (2).Generate normalized autocorrelation function.
- (3).Compute the spectrum $S_y(f)$ as a DFT of the autocorrelation function.
- (4).Compute transfer function $H(f)$
- (5).Compute $Y(f)$, as a product of $H(f)$ and $X(f)$.
- (6).Execute the IDFT of $Y(f)$ to obtain $y(n)$.

Fig.1 shows the block diagram of FCG. The linear convolution of $h(n)$ and $x(n)$ are done by taking care of the circular convolution property in DFTs.

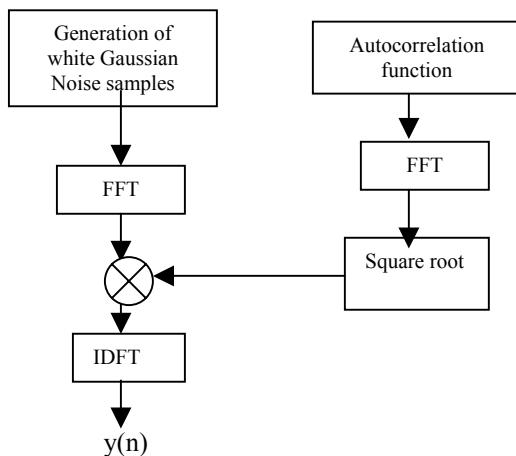


Fig.1 Generation of random samples $y(n)$ with Gaussian autocorrelation function

3.Evaluation of FCG :

The correctness of the generator is verified by the comparison between theoretical and simulated values. The correlation coefficient and average power are estimated and compared with theoretical values. The

autocorrelation coefficient with several temporal delays is estimated from the following estimator

$$\hat{\rho}(m) = \frac{\sum_{i=1}^N x_i^*(1)x_i(m+1)}{\sqrt{\sum_{i=1}^N |x_i(1)|^2 \cdot \sum_{i=1}^N |x_i(m+1)|^2}}$$

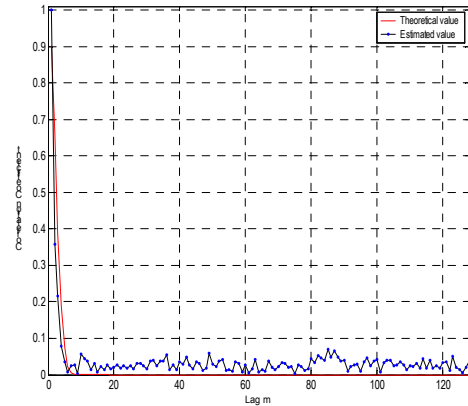


Fig.2.Modulus of the estimated and theoretical correlation coefficient versus lag m . $\lambda=0.05501m$, $PRT=0.001s$, $V_D=0m/s$, $\sigma_v=2m/s$, $N_P=128$, $N=1000$, $\rho(1)=0.9008$.

Where $m = 0, \dots, N_P$ (Number of samples) and N is the number of time series. The correlation coefficient is as shown in Fig.2.

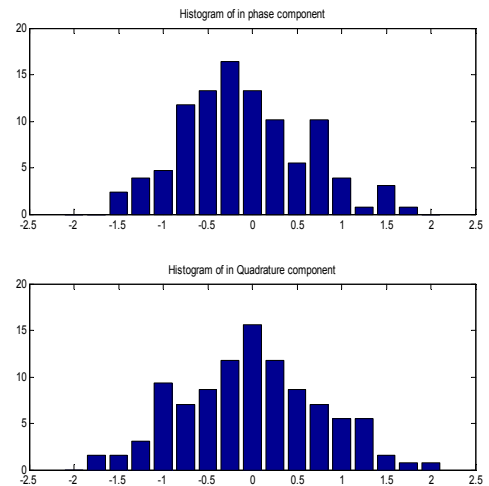


Fig.3.Histogram of in phase and in Quadrature components indicating the Gaussian distribution.

If P is the power that has to be estimated, then the variance of power is given as

$$\text{var}(\hat{P}) = \frac{P^2}{M^2} \cdot \sum_{m=-(M-1)}^{(M-1)} (M - |m|) \cdot |\rho(m)|^2$$

Fig.4 shows the behavior of the standard deviation of average power at the receiver (Square Law) :

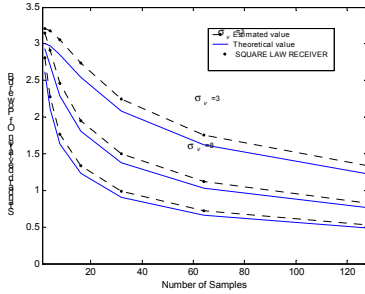


Fig.4 Standard deviation of power

4. FCG for HV polarization pair generation:

The FCG described in the above sections is used to generate two time series y_H and y_V obtained for horizontal and vertical polarization, which must have an assigned correlation coefficient,

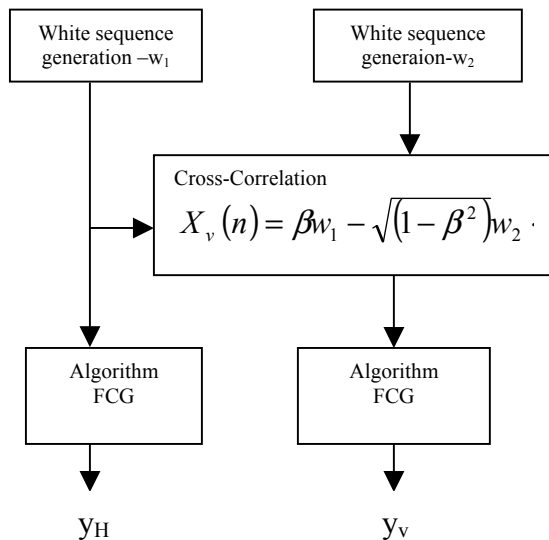
$$\rho_{HH}(n) = \rho_{VV}(n) = \rho(n)$$

and an assigned cross-correlation given by

$$\rho_{HV}(n) = \rho_{HV}(0) \cdot \rho(n)$$

where $\rho_{HV}(0)$ is the correlation (term by term) between the sequences y_H and y_V . The flow char to generate HV polarization pair is shown in Fig.5

Fig.5 FCG for HV polarization pair $\beta = \rho_{HV}(0)$



Differential reflectivity Z_{DR} is a quantity of considerable interest in radar meteorology. The estimator for Z_{DR} is given as

$$\hat{Z}_{DR} = \frac{\frac{1}{M} \sum_{i=1}^M |H(i)|^2}{\frac{1}{M} \sum_{i=1}^M |V(i)|^2} = \frac{\hat{P}_H}{\hat{P}_V}$$

where M is the number of sample pairs $H(i)$ and $V(i)$, the variance is estimated as

$$\text{var}(\hat{Z}_{DR}) = 2Z_{DR}^2 (1 - |\rho_{HV}(0)|^2) \sum_{m=-(M-1)}^{(M-1)} \frac{(M - |m|)}{M^2} |\rho(m)|^2$$

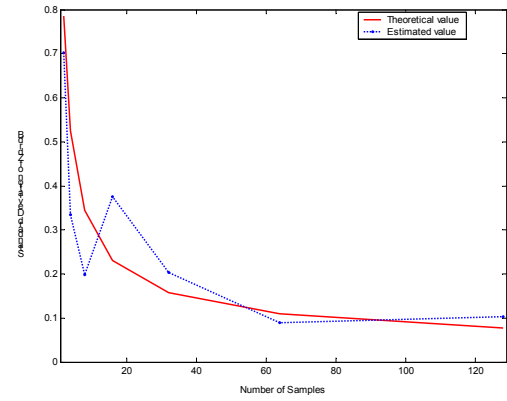


Fig 6. Standard deviation of Z_{DR} in dB

5. Frequency Domain Method: An overview:

This part of the project focuses on the frequency domain algorithm for generation of Dual Polarized echoes, and concentrate on implementation of oversampling . Here primarily the motivation is to generate the time series corresponding to a specified correlation structure. As mentioned earlier, once a pulse of width T_o is sent through the atmosphere or precipitation medium, the echo signal is a result of scattering from a large collection of hydrometeors packed in a close volume. In general, precipitation is composed of a large number if hydrometeors extending over a large range with widely different scattering amplitudes moving with different velocities. The received voltage can also be expressed as a discrete sum

of the contribution from individual particles in the medium as,

$$V_r(t) = \sum_k A_K(\tau_k; t) e^{-j2\pi f_o \tau_k} U_{tr}(t - \tau_k)$$

where the symbols bear their usual significance. As part of the sampling scheme the radar transmits a pulse of width T_o and then samples the returns over an interval T_S , i.e. the Pulse Repetition time. If the sampling interval is small enough then there will be significant correlation between the returns from the medium. Or in other words a narrow distribution of velocities among closely spaced particles can be attributed to this correlation.

When a pulse of duration T_o is transmitted the signal power received at time t is due to precipitation particles at a resolution volume located from radar at a distance given by $ct/2$. Now, as a sequence of pulses spaced T_S in time are transmitted, observations of $V_r(t), \dots, V_r(t + mT_S)$ provide temporal samples of received signal at the aforesaid distance. Now if the scatterers are uniformly distributed in space the contribution over a period can be broken up into contributions from sub-volumes, or alternatively viewed as contribution coming from the concerned resolution volume during a fraction of the previous sampling period. Each elemental resolution volume can be associated with an equivalent distance. If T_o used to be the previous sampling period, now the sampling time is T_o/L , where L refers to the oversampling factor. Therefore, $cT_o/2L$ happens to be present range resolution since the pulse propagates through a distance $cT_o/2L$ during the subpulse interval T_o/L . This can be interpreted as evoking over-sampling in range by taking more number of samples in a given period, providing us more information about the continuously changing behaviour of the precipitation medium at various ranges. In order to see the fluctuation that takes place over the passage of dispersed pulse, we simulate the signal characteristics at a time-scale corresponding to propagation time over a resolvable range bin.

6. Characterization of Dual Polarized weather echoes:

For Dual Polarized echoes, two time series corresponding to the polarization states for each range are simulated. Certain assumptions are made in this process:

1. The in phase and quadrature component of the complex echo signal are independent, gaussian random variables with zero mean.
2. The Power Spectrum of weather echoes is a power weighted distribution of radial velocities of all scatters in the resolution volume, and the sum, or the resultant echo can be modeled as having a Gaussian distribution, courtesy- Central Limit Theorem.

H and V signals together form a complex bivariate Gaussian process, so to simulate them we need to specify certain joint time correlation and cross correlation properties. Say the spectral coefficients for N samples of a bivariate time series are given by,

$$S(k) = \begin{bmatrix} S_H(k) \\ S_V(k) \end{bmatrix} = \begin{bmatrix} a_H(k) + jb_H(k) \\ c_H(k) + jd_H(k) \end{bmatrix}$$

Given the wavelength, mean velocity, spectral width, reflectivity and SNR, we can constitute the co-spectrum as follows.

$$f_{HH, VV} = \frac{S_{H,V}}{\sqrt{2\pi\sigma_f}} e^{-(f_k - \bar{f})^2 / 2\sigma_f^2} + NT_s$$

Where $\bar{f} = 2\bar{v} / \lambda$ is mean Doppler frequency, $\sigma_f = 2\sigma_v / \lambda$ is spectral width. S_H is the mean signal power in horizontal polarization, N is white noise power spectral density and T_s is sample spacing.

Further assuming $\rho_{HV}(m) = \rho(m)\rho_{HV}(o)$ the cross spectrum can be expressed as,

$$f_{HV}(k) = c_{HV}(k) - jd_{HV}(k) = f_{HH}(k)\rho_{HV}(o)$$

The covariance matrix of (H,V) can be written as,

$$C = \begin{bmatrix} f_{HH}(k) & 0 & c_{HV}(k) & d_{HV}(k) \\ 0 & f_{HH}(k) & -d_{HV}(k) & c_{HV}(k) \\ c_{HV}(k) & -d_{HV}(k) & f_{VV}(k) & 0 \\ d_{HV}(k) & c_{HV}(k) & 0 & f_{VV}(k) \end{bmatrix}$$

As real and imaginary components of H&V signals are independent, we obtain the vector [S]

containing the spectral coefficients using the following equations:

$[C] = [T]^T [T]$ And $S = XT$ where X is 1×4 iid Normal (0,1) vector. At this point, the time series for a particular range bin can be found by computing the IFFT of spectral sequence.

7. Generation of Oversampled Signal samples:

The following diagram shows how individual samples are combined to form signal at the receiver input.

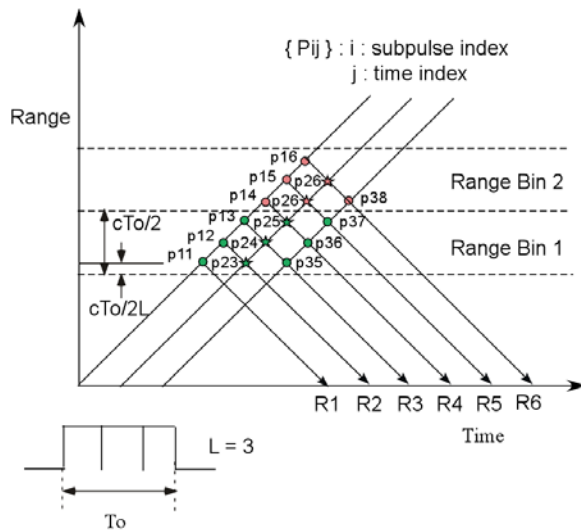


Fig.7: Illustration of Oversampling process

As is obvious from the figure, a time series is simulated for each range, and then depending on the sampling instant (at receiver) the elements in the time series(s) are combined to form input at the receiver. For example, In the above diagram, $\{R_i\}$ refers to the receiver samples where,

$$R1 = p11; R2 = p12 + p23; R3 = p13 + p24 + p35;$$

$$R4 = p14 + p25 + p36;$$

As the first subpulse/chip is propagating, at the first range-sampling instant it hits the first range resulting in p11. At the next range-sampling instant we have two echoes, one from the 2nd chip hitting the first range bin (p23) and the other (p12) due to first chip which has propagated through a fractional distance of first range bin ($1/L$). If L is the oversampling factor, and N is number of samples/PRT, then during oversampling we have NL samples /PRT at receiver and each simulated time series(s) has $2NL$ samples/PRT. It can be seen from the diagram that the individual echoes who combine to form the receiver input are spaced at an interval half that of the receiver's sampling interval (To/L). The first chip gives rise to the echo p11 and keeps propagating. In the time interval $To/2L$ it

propagates through a distance $cTo/2L$ and the echo from the range comes back to combine with the 2nd chip which hit the first range resulting in p23 after an interval of To/L since the first chip encountered the first range bin.

The voltages $R(i)$ are identically distributed complex Gaussian random variables. Their distribution properties are illustrated in the following figures.

Fig. 8: Histogram of real part of received signal

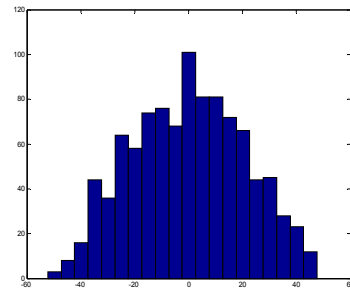


Fig.9: Histogram of imag. part of received signal

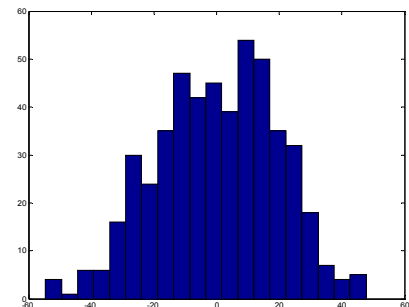
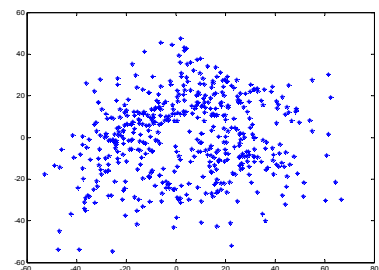


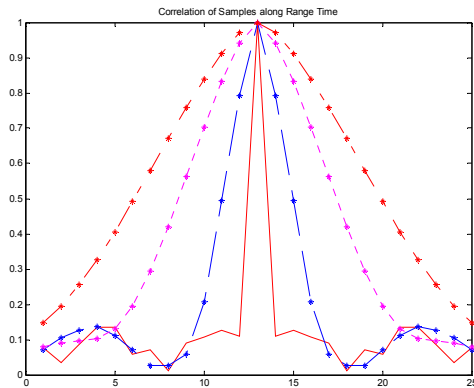
Fig.10 : Scatter plot



For a rectangular transmitted pulse and assuming infinite bandwidth, the correlation coefficient of samples along range time can be expressed as,

$$\rho^{(R)}(m) = \begin{cases} \frac{L - |m|}{L} & |m| < L \\ 0 & \text{otherwise} \end{cases}$$

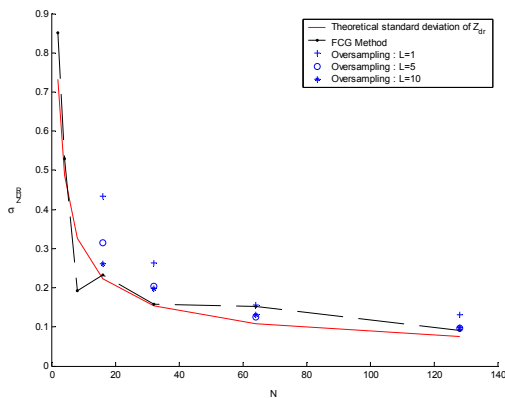
Fig.11: Range Time Autocorrelation function for $L=1,3,7\&10$



8. Comparative study of two algorithms:

It is observed that the FCG method is more computationally efficient and gives accurate estimates at low value of N . Whereas the Frequency domain based algorithm's performance degrades as N decreases, but again, as shown in Fig.12 when oversampling is done, it's performance becomes comparable with that of FCG method even at low values of N . Moreover it turns out that the frequency domain based method is more general as it can simulate bivariate Gaussian random process with arbitrary auto and cross correlation.

Fig.12 Standard Deviation of ZDR estimated using both FCG & Frequency Domain methods:



9. References:

1. Statistical properties of dual polarized radar signals – Chandrasekar V. & V.N.Bringi -1986
2. Polarimetric Doppler Weather Radar – Principle and applications: Bringi & Chandrasekar – 2001
3. Estimation of Doppler and Polarimetric variables for weather radars – S.M. Torres - 2001
4. Computer simulation of weather radar signals – G. Galati & G. Pavan
5. Pulse Compression of Weather Radars – Mudukutore et al -1998