A little E&M review

• Coulomb’s Law
• Gauss’s Law
• Ohm’s Law
Coulomb’s Law

Interaction of two point charges, electrostatic attraction between two point charges (of opposite sign)

\[ F = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2} \]

If \( F \) is divided by \( q \), the Electric Field is defined.

\[ \frac{F}{q_1} = \frac{1}{4\pi\varepsilon(q_2)} / r^2 \]

\[ E = \frac{1}{4\pi\varepsilon(q)} r^{-2} \]

E field around point charge of value q.
**Gauss’s Law:** The total electric flux through a closed conducting surface is equal to the total net charge on the surface. In other words, if a closed surface of any shape is constructed in a region in which an E field is present, the surface integral of the normal component of the E field over the surface area is equal to the net charge on the surface divided by the permittivity.

\[ \oint E \cdot \hat{n} \, dA = \frac{Q}{\varepsilon_0} \]

Gauss’s Law in differential form is

\[ \nabla \cdot E = \frac{\rho}{\varepsilon_0} \]

And in one dimension...

\[ \frac{\partial E}{\partial z} = \frac{\rho}{\varepsilon_0} \]

Charge density may be inferred from the vertical gradient of E.
Balloon borne Electric Field Meter—EFM (Winn and Byerley (1975; QJRMS))

Two 15 cm hollow copper spheres, separated by 2 cm. The spheres rotate at 2.5 Hz. The spheres also contain a 400 MHz radiosonde transmitter, and various circuits. The spheres themselves act as the antenna for the transmitter. The in-cloud electric field causes opposite polarity of charge to develop on the two spheres which causes a sinusoidal signal at the spin rate. A mercury switch in the tube between the spheres switches on and off to determine which sphere is up. From that the sign of $E_z$ can be determined. Charge density can then be derived by using Gauss’ Law in the vertical dimension. The entire assembly also rotates in the horizontal allowing $E_h$ to be determined. A conventional radiosonde is flown a few meters below the EFM to measure thermodynamics.

Fig. 6.9. Updated version of balloon-borne electric field meter, EFM, originally designed by Winn Byerley (1975). The aluminum spheres are the sensors of the electric field and the transmitting antenna. Electronics are housed in one sphere and their battery power source in the other. The dimensions shown are typical, but not critical. The vanes are low-density styrofoam. (From Marshall et al. 1995b: American Geophysical Union, with permission.)
Shaded layers denote layers with negative charge. Non-shaded, positive charge.
Ohm’s Law

\[ J = \sigma E \]

\( J \) is the current density, coulombs/m\(^2\)

For the Earth’s atmosphere, \( J \) is \( 2.7 \times 10^{-12} \) A m\(^{-2}\) (\( J \) constant with height in fair weather). This is the air earth current.

Here the symbol \( \sigma \) is the conductivity with units of S/m, Siemens/meter, of Mhos.

Conductivity is a measure of the ease which charge moves through a given medium.

Ohm’s Law may also be expressed in simple “circuit” form, \( V=IR \) where \( V \) is the voltage, \( I \) is the current and \( R \) is the resistance (Ohms).
What is the value of the conductivity near the Earth’s surface? Assume \( E \) is 150 V/m.

\[
\sigma = \frac{J}{E} = \frac{2.7 \times 10^{-12}}{1.5 \times 10^2} \approx 2 \times 10^{-14} \text{ Mhos}
\]

This conductivity is equivalent to the conductivity of porcelain, which is used as an insulator. Sigma is sometimes replaced with the symbol \( g \).

Conductivity increases with height owing to cosmic ray ionization. Near the Earth’s surface conductivity is low as ions are attached to “bulky” aerosol particles, reducing the mobility of the ions and thus decreasing the conductivity.
Voltage difference is approximately 300 kV

So we live in a giant capacitor. Earth’s surface is the lower plate and the electrosphere about 50-70 km overhead is the upper plate. Upper plate is positively charged and the lower plate is negatively charged. The voltage difference across the capacitor is about 300 kV. What is the voltage difference across an individual 2 meters tall standing at the Earth’s surface?
The capacitance $C$ of a concentric spherical capacitor is given by:

$$C = 4\pi \varepsilon_0 \frac{r_1 r_2}{r_2 - r_1}$$

So what is the capacitance of the Earth-electrode spherical capacitor? Assume $r_2$ is $r_1$ plus 50 km.

$$C = 4\pi 8.85 \times 10^{-12} \times 6.37 \times 6.420 \times 10^{12} / 2.5 \times 10^4$$

$C = 0.182$ Farads

Measurements of the charge density on the Earth’s surface yields a value of $3.2 \times 10^{-9}$ Cm$^{-2}$.

So the total charge on the Earth’s surface is $3.2 \times 10^{-9}$ Cm$^{-2} \times$ Area of Earth $= 1.65 \times 10^6$ C
WHAT IS THE ENERGY STORED IN THIS CAPACITOR?

Energy \( = \frac{Q^2}{2C} \)

Plugging in numbers gives \( 7 \times 10^{12} \) Joules

This is the energy of a million 12 V 100 Amp-hr automobile storage batteries.
A little history

• About 1860, Lord Kelvin developed a theory for the global circuit; this was about 40 years before Kennely and Heaviside postulated the ionosphere (Marconi verified the “Heaviside” layer by being the first to carry our trans-Atlantic radio communications.

• Lord Kelvin’s work motivated surface measurements of the fair weather electric field, including the Carnegie and Maud cruises. His work also verified that the Earth’s surface carries net negative charge.

• Early 1900’s, CTR Wilson measured E field changes associated with thunderstorms and determined that thunderstorms systematically have positive charge in their upper regions and negative charge in their lower regions. He also proposed that thunderstorms are batteries driving the global circuit. He also suggested that “shower” clouds contribute to the global circuit by transporting negative charge downward on precipitation particles.
The Global Circuit. The producer of the global circuit is global thunderstorm activity. The consumer of the global circuit is fair weather electricity, that is, current flow from the upper electrosphere to the Earth’s surface. The global circuit is beautifully described by one scalar value, the potential difference between the Earth’s surface and the ionosphere.
Diurnal variation of the fair weather E field

From MacGorman and Rust.

The famous “Carnegie” curve, which is the diurnal variation of the fair weather electric field measured over the open ocean. The left panel plots estimates of the diurnal variability of thunderstorm area over the 3 “tropical” chimney regions.
We have seen that the Earth’s concentric spherical capacitor is a “leaky” dielectric, That is the conductivity of the atmosphere is finite. Without the action of Thunderstorms that transfer net positive charge to the upper electrode and lower net negative charge to the lower electrode, the fair weather electric field would run down over time. We can estimate the e-folding time for the fair weather electric field.

\[ E(t) = E_0 e^{-\tau/RC} \]

Here RC is the so-called time constant, the product of the atmosphere’s resistance and its capacitance.
Combining the following equations......

\[ E \cdot A = Q / \varepsilon \]

\[ C = Q / \Delta U \]

\[ J = \sigma E \]

\[ I = \sigma EA \]

Therefore

\[ RC = \Delta UC / \sigma EA = \Delta U Q / \sigma EA \Delta U = EA \varepsilon / \sigma EA = \varepsilon / \sigma \]

\[ \varepsilon = 8.85 \times 10^{-12} \]

\[ \sigma = 10^{-14} \]

**e-folding time is about 15 minutes!!**
What is current flow in the global circuit?

• We can find the current flowing in the global circuit quite easily.

\[ I = J \cdot 4 \pi R_E^2 \]

\[ I = 2.7 \times 10^{-12} \text{ A/m}^2 \cdot 4\pi(6.378 \times 10^6)^2 \]

\[ I \approx 1.4 \text{kA} \]

• What is the “power” in the global circuit?

• Power (P) = V x I \ (300 \text{ kV} \times 1400 \text{ A}) = 4.2 \times 10^8 \text{ W}